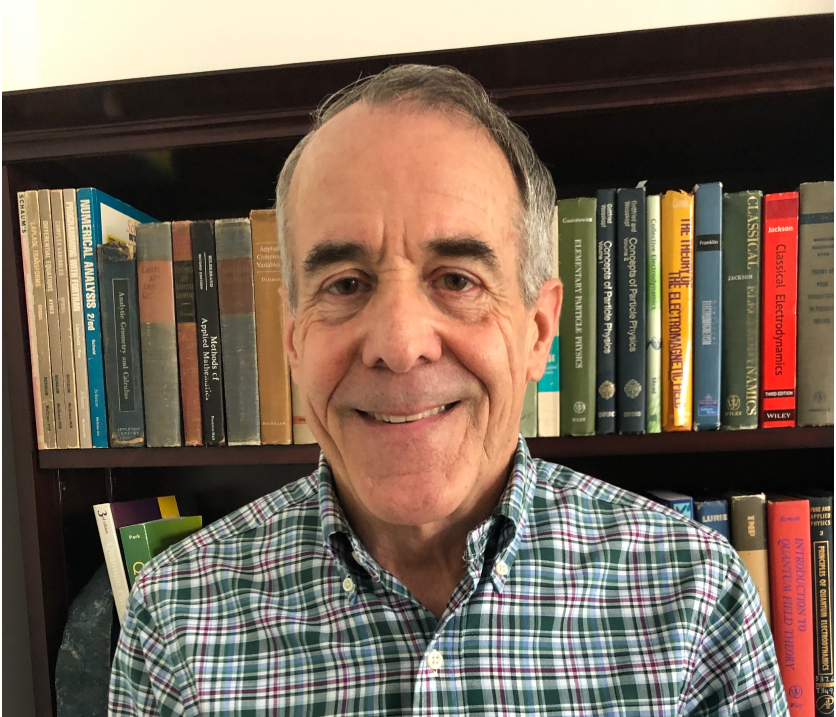
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PROPERTIES OF THE QUANTUM VACUUM CALCULATED FROM ITS STRUCTURE

G. B. Mainland and Bernard Mulligan

Department of Physics

The International Association for Relativistic Dynamics
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CENTRAL IDEAS OF TALK

- The properties of the quantum vacuum can be calculated from its structure and appropriate field equations.

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- Vacuum fluctuations spontaneously appear in – and disappear from – the vacuum. While present, vacuum fluctuations can be polarized by photons, slowing their progress similarly to the way that a normal dielectric slows photons.

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- The interaction of photons with vacuum fluctuations determines the permittivity of the vacuum, the speed of light in the vacuum, and the fine-structure constant.

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- The interaction of photons with vacuum fluctuations determines the permittivity of the vacuum, the speed of light in the vacuum, and the fine-structure constant.
- Properties of vacuum fluctuations explain why the huge vacuum energy density cannot create a gravitational force or contribute to the cosmological constant.

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- Physicists use the term "vacuum fluctuation" to describe two very different entities:
- Type 1 vacuum fluctuations can interact with normal (meaning real, ordinary, and observable) test particles placed in the vacuum, but only in very limited ways.
- Type 2 vacuum fluctuations have no observable consequences.

TYPE 1 VACUUM FLUCTUATIONS

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- Field theory provides a proof that type 1 vacuum fluctuations must exist as fluctuations of free fields.

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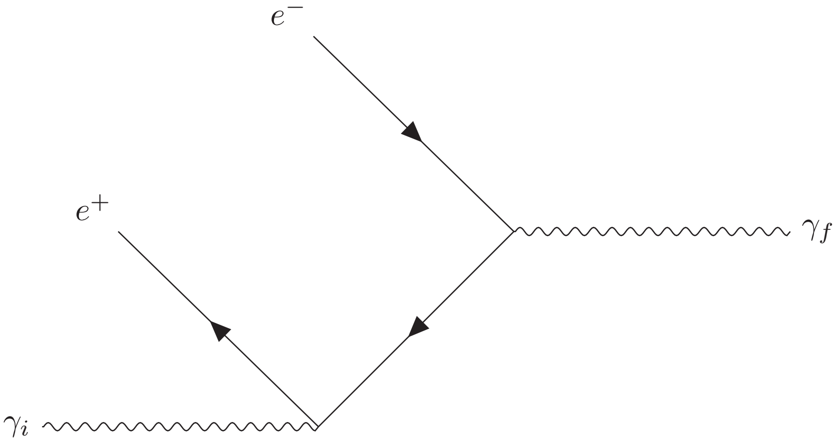
- If the particle and antiparticle are massive, the pair must appear with zero center-of-mass momentum in the least energetic bound state.
- Massive particle-antiparticle vacuum fluctuations appear as transient, spin-0 atoms in their lowest energy state.

FEYNMAN DIAGRAM OF A TYPE 1 ELECTRON-POSITRON VACUUM FLUCTUATION INTERACTING WITH A PHOTON



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- cannot exert a force on normal matter.
- If they could exert a force, they could do work on normal particles. Then when a vacuum fluctuation vanished back into the vacuum, the energy associated with any work done by the vacuum fluctuation would be left behind in the physical world, violating conservation of energy.

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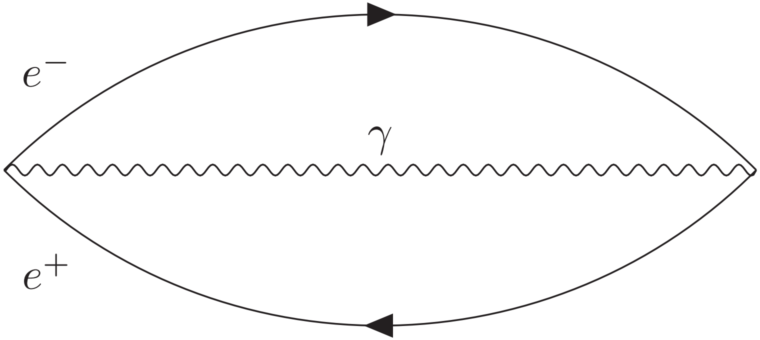
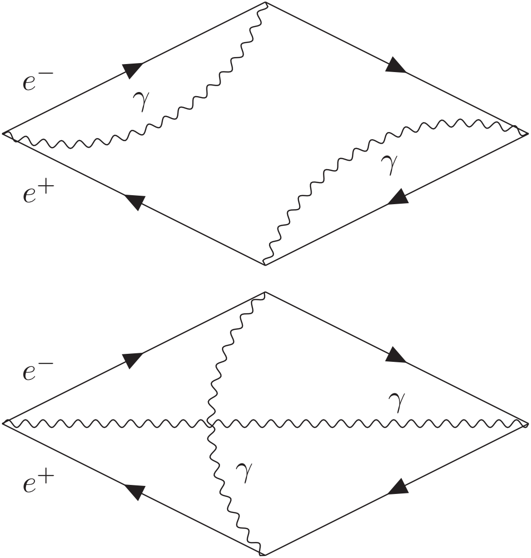
FEYNMAN DIAGRAM FOR A TYPE 2 VACUUM FLUCTUATION: (VACUUM DIAGRAM OR VACUUM BUBBLE)



FEYNMAN DIAGRAMS FOR TYPE 2 VACUUM FLUCTUATIONS



ELECTROMAGNETIC PROPERTIES OF THE VACUUM

- It should be possible to calculate the electromagnetic properties of the vacuum from the structure of the (quantum) vacuum and Maxwell's equations. What are the electromagnetic properties of the vacuum?

ELECTROMAGNETIC PROPERTIES OF THE VACUUM

PARAMETER	PROPERTY OF VACUUM & QED
permittivity ϵ_0 of vacuum	YES

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The value of μ_0 was originally arbitrarily chosen so that the size of the unit of current was equal in two different systems of units: $\mu_0 \equiv 4\pi \times 10^{-7}$ H/m. μ_0 is a measurement-system constant, not a property of the vacuum.

ELECTROMAGNETIC PROPERTIES OF THE VACUUM

PARAMETER	PROPERTY OF VACUUM & QED
permittivity ϵ_0 of vacuum	YES
permeability μ_0 of vacuum	NO
speed $c = 1/\sqrt{\mu_0 \epsilon_0}$ of light in the vacuum	YES

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The fine-structure constant α is given by

$$\alpha \equiv \frac{e^2}{4\pi\epsilon_0 \hbar c} ,$$

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PARAMETER	PROPERTY OF VACUUM & QED
permittivity ϵ_0 of vacuum	YES
magnetic constant μ_0	NO
speed $c = 1/\sqrt{\mu_0 \epsilon_0}$ of light in the vacuum	YES
fine structure constant α	YES

THEORETICAL FORMULA FOR ϵ_0

In the vacuum Maxwell's equation $\nabla \cdot \mathbf{D} = \rho$ implies the relation

$$\epsilon_0 \mathbf{E} = \mathbf{P}\,,$$

where $\mathbf{D}$ is the electric displacement, ρ is the charge density, and $\mathbf{P}$ is the polarization density.

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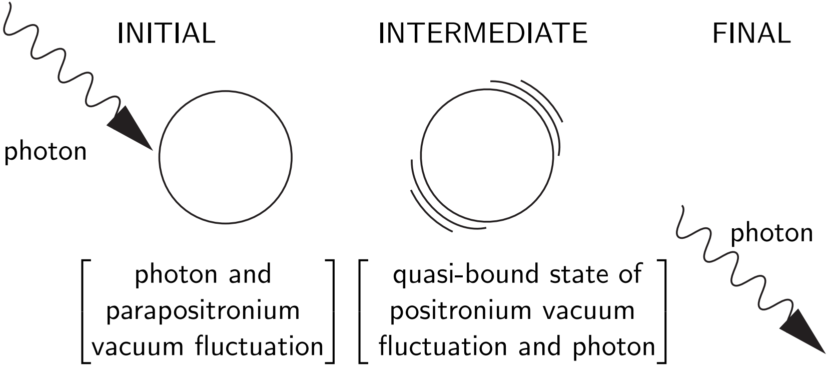
The polarization density for a vacuum fluctuation is calculated somewhat similarly to the way it is calculated for a dielectric consisting of normal matter.

PHOTON IN THE VACUUM POLARIZING A PARAPOSITRONIUM VACUUM FLUCTUATION



THEORETICAL FORMULA FOR ϵ_0 CONTINUED

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- The polarization density for a vacuum fluctuation is calculated similarly to the way it is calculated for a normal dielectric.
- The total polarization density $\mathbf{P}$ from the three types of charged lepton-antilepton is
  - $\epsilon_0 \mathbf{E} = \mathbf{P} \cong 3 \frac{(2^7)}{\pi} \frac{e^4}{(\epsilon_0 \hbar c)^2} \mathbf{E}$

THEORETICAL FORMULA FOR ϵ_0 COMPLETED

Equating the first and last terms in the equation on the previous slide and cancelling the electric field $\mathbf{E}$ yields

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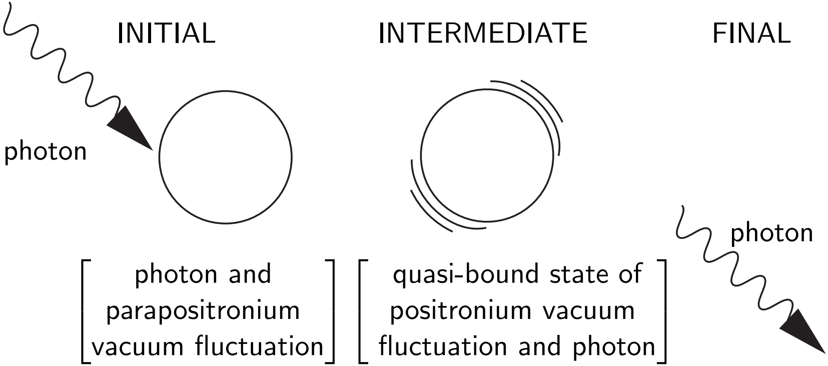
Using $c = 1/\sqrt{\mu_0 \epsilon_0}$ and solving for ϵ_0,

$$\epsilon_{0\ \mathrm{theory}} \cong \frac{6\mu_0}{\pi}\left(\frac{8e^2}{\hbar}\right)^2 = 9.10 \times 10^{-12}\frac{\mathrm{C}}{\mathrm{Vm}}$$

$$\epsilon_{0\ \mathrm{accepted}} = 8.85 \times 10^{-12}\frac{\mathrm{C}}{\mathrm{Vm}}$$

SLOWING OF PHOTONS IN THE VACUUM BY A PARAPOSITRONIUM VACUUM FLUCTUATION



THEORETICAL FORMULA FOR THE SPEED OF LIGHT c IN THE VACUUM

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The calculated value for c is 1.3 % less than the defined value.

THE FINE STRUCTURE CONSTANT α

In 1988 Richard Feynman wrote about the fine-structure constant: "It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.... It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the 'hand of God' wrote that number, and 'we don't know how He pushed his pencil.' "

CALCULATION OF THE FINE STRUCTURE CONSTANT α: "HOW HE PUSHED HIS PENCIL"

The fine-structure constant α is defined by

$$\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c} \,.$$

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The fine-structure constant α is defined by

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Substituting the theoretical formulas for ϵ_0 and c into the above expression for α,

$$\left(\frac{1}{\alpha}\right)_{\text{theoretical}} \cong 8^2\sqrt{3\pi/2} = 138.93\ldots$$

$$\left(\frac{1}{\alpha}\right)_{\text{experimental}} = 137.03\ldots .$$

CONSISTENCY CONDITIONS THE CALCULATION OF c MUST SATISFY

1. "The speed of light in the vacuum has the same value c in every direction in all inertial reference frames." (Postulate 2 from Einstein's 1905 paper "On the Electrodynamics of Moving Bodies" in which he introduced special relativity.)

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- The vacuum is isotropic and at rest with respect to every inertial frame.
- Maxwell's equations are the same in every inertial frame. (Postulate 1 from Einstein's 1905 paper.)
- c is calculated from properties of the vacuum and Maxwell's equations so c is the same in every direction in all inertial frames.

CONSISTENCY CONDITIONS THE CALCULATION OF c MUST SATISFY

2. and 3. The derivation of c used the formula $c = 1/\sqrt{\mu_0 \epsilon_0}$, which is true only in the vacuum where the general relations $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ and $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ become, respectively, $\mathbf{D} = \epsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$.

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- The derivation of ϵ_0 used the relation $\mathbf{D} = \epsilon_0\mathbf{E}$.
- The magnetization $\mathbf{M}$ is zero for particle-antiparticle vacuum fluctuations of massive particles, implying that $\mathbf{B} = \mu_0\mathbf{H}$.

GRAVITATIONAL PROPERTIES OF THE VACUUM: ENERGY DENSITY

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- If there is no cutoff, the energy density is infinite.
- The huge disparity between the calculated value of the vacuum energy density of the universe and the observed energy density is known as the "vacuum catastrophe".

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GRAVITATIONAL PROPERTIES OF THE VACUUM: ENERGY DENSITY

- Because vacuum energy is conserved independently from normal energy, vacuum fluctuations cannot exert forces on normal particles. Thus the vacuum energy density cannot contribute to the normal energy density, implying there is no "vacuum catastrophe".

GRAVITATIONAL PROPERTIES OF THE VACUUM: COSMOLOGICAL CONSTANT

The cosmological constant Λ, which plays a a crucial role in explaining the accelerating rate of expansion of the universe, appears to depend on the energy density $\rho_{\mathrm{vac}}^{\mathrm{energy}}$ of the vacuum,

$$\Lambda = \frac{8\pi G}{c^4}\rho_{\mathrm{vac}}^{\mathrm{energy}} \,, \tag{1}$$

where G is the gravitational constant.

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- Eq. (1) is obtained after assuming that the vacuum energy density can exert a gravitational force.
- Since the vacuum energy density cannot exert a gravitational force, $\rho_{\mathrm{vac}}^{\mathrm{energy}}$ does not contribute to Λ as indicated by (1), resolving "the old cosmological constant problem".