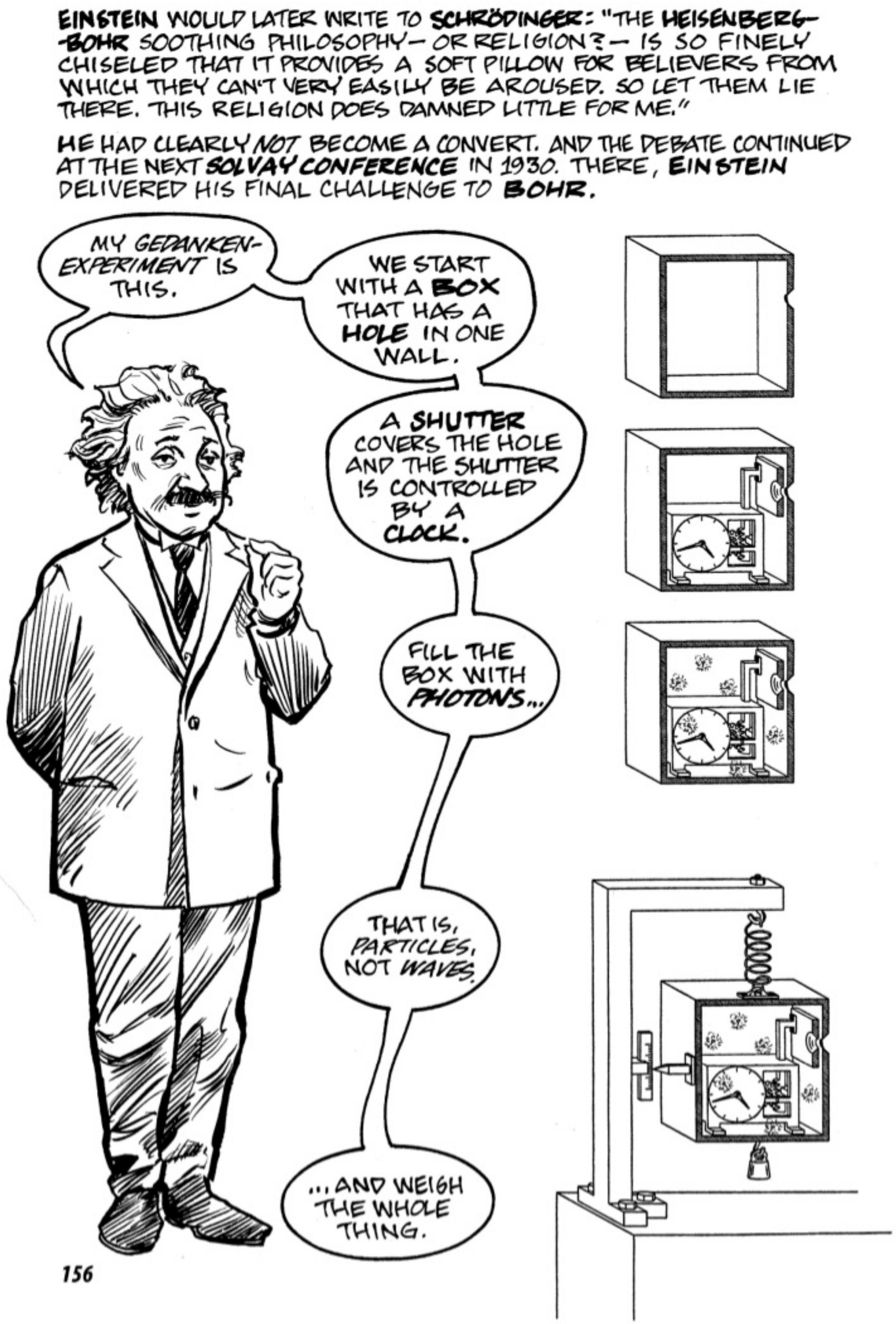
Time Dispersion in Time-of-Arrival

John Ashmead

Can we prove that the Heisenberg uncertainty principle does not apply along the energy/time axis in the same way it applies along the space/momentum axis?

International Association for Relativistic Dynamics - 2020 Virtual



Heisenberg Uncertainty Principle (HUP)

- $\Delta E \Delta \tau \geq 1$ taken for granted
- Critical in early days of QM
- But basically abandoned since
- $\imath \frac{\partial}{\partial \tau}\psi_\tau(x) = -\frac{1}{2m}\frac{\partial^2}{\partial x^2}\psi_\tau(x)$

Three ways of saying the same thing

$$\Delta E \Delta t \geq 1$$

- Heisenberg uncertainty principle applies in energy/time

$$\psi_\tau(x) \to \psi_\tau(t,x)$$

- Wave function extends in time (or energy)

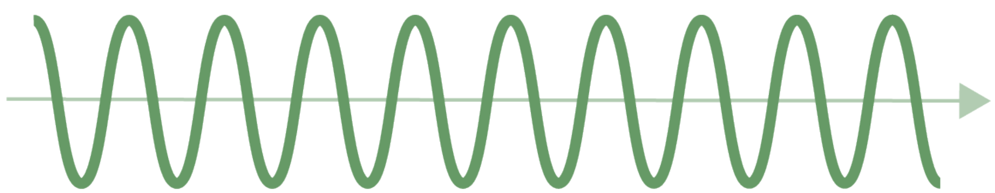
$$\hat{E} = \imath \frac{\partial}{\partial t}$$

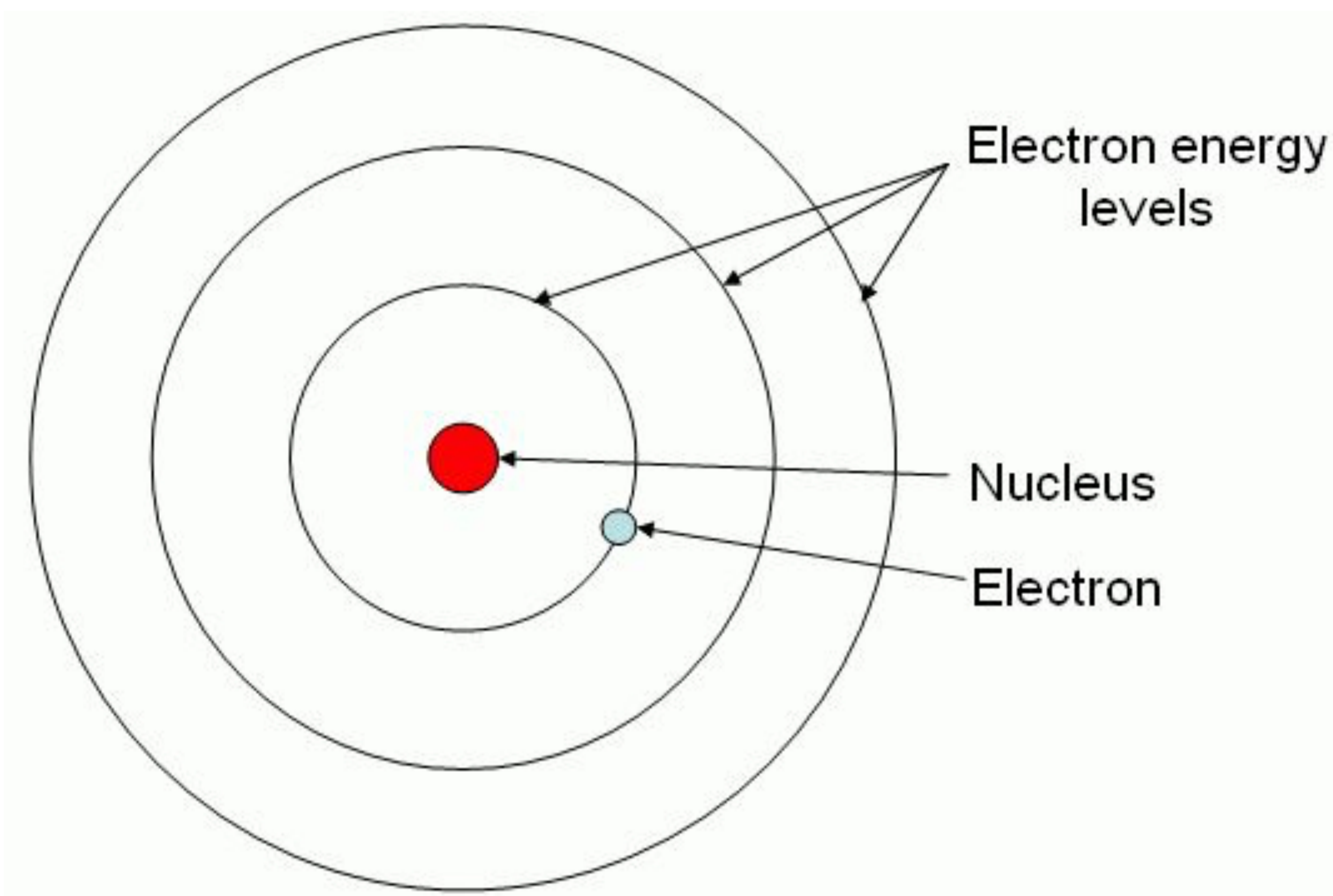
- Time is an observable

How to get to Falsifiability?

- Look at the class of theories, not just one particular theory
- Use the simplest possible approaches
- Look for order of magnitude estimates
- Use covariance to eliminate need for free parameters

Why haven't we seen this before?





- Time for photon to cross atom?
- .35 Attoseconds
- Explains why we haven't seen

Can we see it now?

Attosecond correlation dynamics

M. Ossiander[1,2★], F. Siegrist[1,2], V. Shirvanyan[1,2], R. Pazourek[3], A. Sommer[1], T. Latka[1,2], A. Guggenmos[1,4], S. Nagele[3], J. Feist[5], J. Burgdörfer[3], R. Kienberger[1,2] and M. Schultze[1,4★]

Photoemission of an electron is commonly treated as a one-particle phenomenon. With attosecond streaking spectroscopy we observe the breakdown of this single active-electron approximation by recording up to six attoseconds retardation of the dislodged photoelectron due to electronic correlations. We recorded the photon-energy-dependent emission timing of electrons, released from the helium ground state by an extreme-ultraviolet photon, either leaving the ion in its ground state or exciting it into a shake-up state. We identify an optical field-driven d.c. Stark shift of charge-asymmetric ionic states formed after the entangled photoemission as a key contribution to the observed correlation time shift. These findings enable a complete wavepacket reconstruction and are universal for all polarized initial and final states. Sub-attosecond agreement with quantum mechanical *ab initio* modelling allows us to determine the absolute zero of time in the photoelectric effect to a precision better than 1/25th of the atomic unit of time.

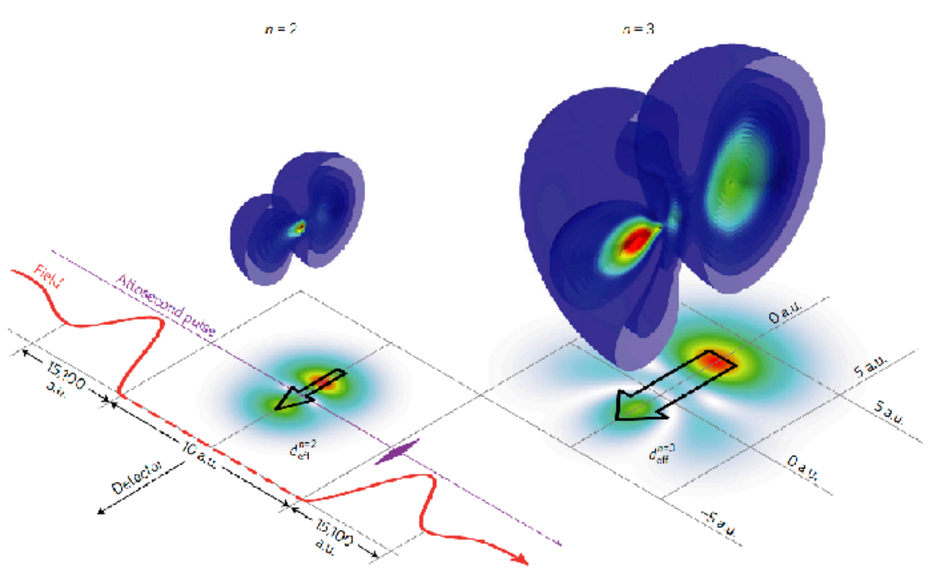


Table 1 | Time shift $\Delta\tau$ between direct and shake-up photoemission in helium for different photon energies.

Photon energy (eV)	$\Delta\tau$ experiment (10^{-18} s)	Standard error (10^{-18} s)	$\Delta\tau$ theory (10^{-18} s)
93.9	−12.6	0.99	−12.7
97.2	−10.6	0.85	−11.0
108.2	−5.0	1.01	−5.9
113.0	−4.9	1.60	−5.8

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Time dispersion in quantum mechanics

John Ashmead[1]

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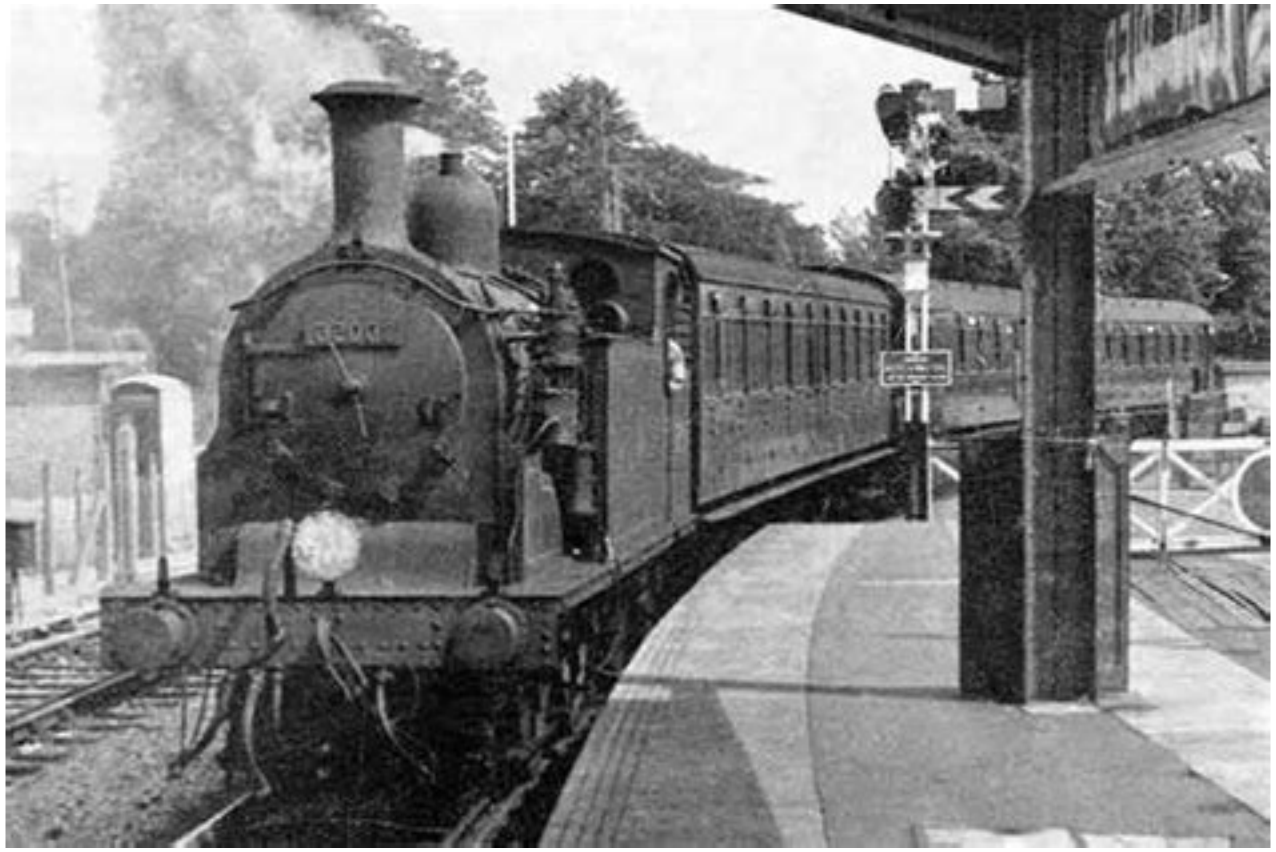
Abstract

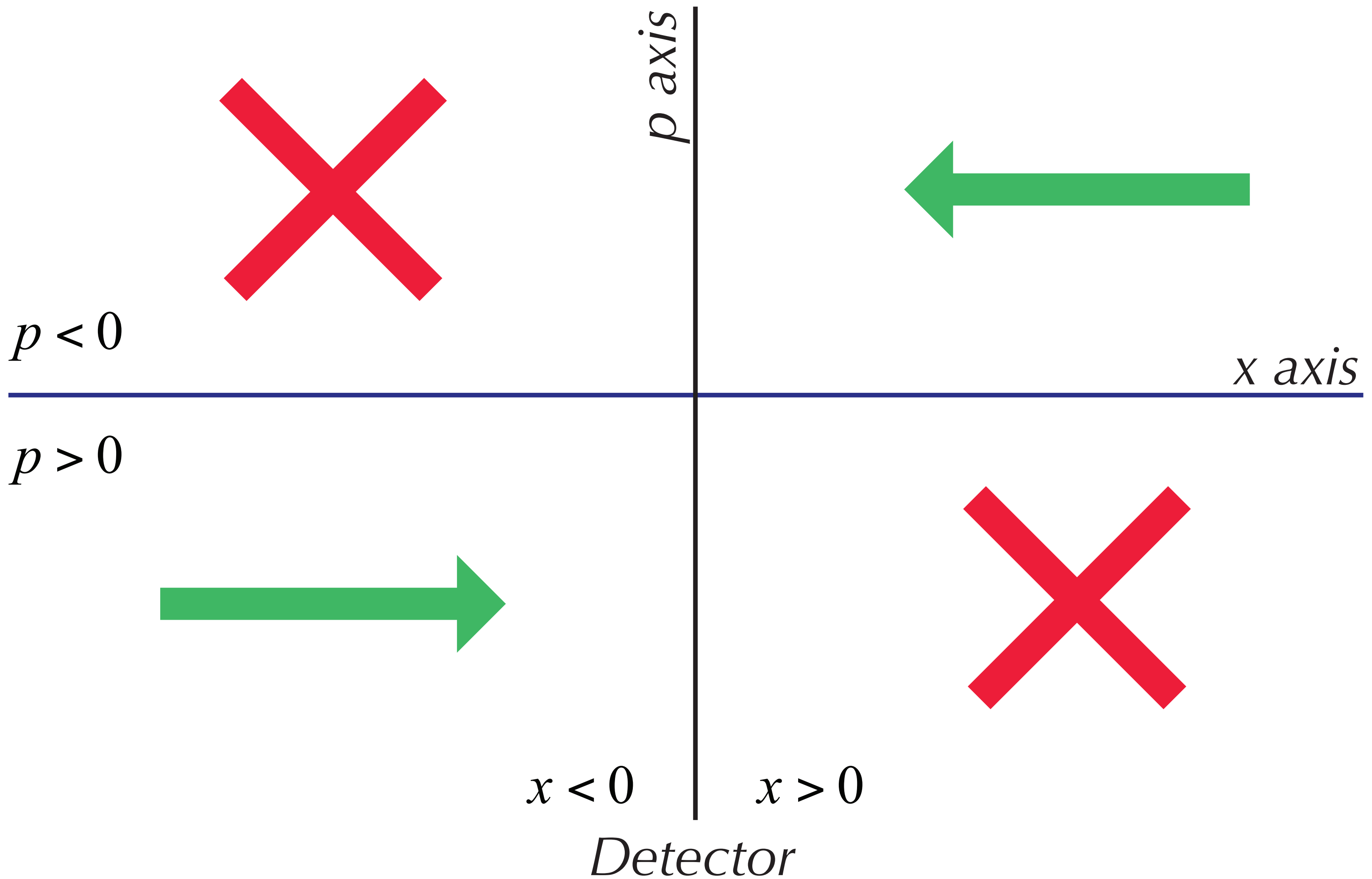
In quantum mechanics the time dimension is treated as a parameter, while the three space dimensions are treated as observables. This assumption is both untested and inconsistent with relativity. From dimensional analysis, we expect quantum effects along the time axis to be of order an attosecond. Such effects are not ruled out by current experiments. But they are large enough to be detected with current technology, if sufficiently specific predictions can be made. To supply such we use path integrals. The only change required is to generalize the usual three dimensional paths to four. We predict a large variety of testable effects. The principal effects are additional dispersion in time and full equivalence of the time/energy uncertainty principle to the space/momentum one. Additional effects include interference, diffraction, and entanglement in time. The usual ultraviolet divergences do not appear: they are suppressed by a combination of dispersion in time and entanglement in time. The approach here has no free parameters; it is therefore falsifiable. As it treats time and space with complete symmetry and does not suffer from the ultraviolet divergences, it may provide a useful starting point for attacks on quantum gravity.

Time of Arrival



- Most obvious test
- Also part of most experiments: there is almost always a time measurement, even if not recorded
- Intrinsic uncertainty in time competes with extrinsic uncertainty
- Curiously, time-of-arrival even in standard quantum mechanics not well understood

"A common theme is that classical mechanics, deterministic or stochastic, is always a fundamental reference on which all of these approaches [to the time-of-arrival operator] are based." – Muga and Leavens [35]



$$\rho_d(\tau) = \left| \int_0^\infty dp \sqrt{\frac{p}{2\pi m}} e^{-\imath \frac{p^2 \tau}{2m}} \varphi^{(left)}(p) \right|^2 + \left| \int_{-\infty}^0 dp \sqrt{\frac{-p}{2\pi m}} e^{-\imath \frac{p^2 \tau}{2m}} \varphi^{(right)}(p) \right|^2 \quad (2.1)$$

Gaussian test functions

$$\varphi_\tau(x) = \sqrt[4]{\frac{1}{\pi\sigma_x^2}}\sqrt{\frac{1}{f_\tau^{(x)}}}\,e^{\imath p_0 x - \frac{1}{2\sigma_x^2 f_\tau^{(x)}}\left(x-\bar{x}_\tau\right)^2 - \imath\frac{p_0^2}{2m}\tau}$$

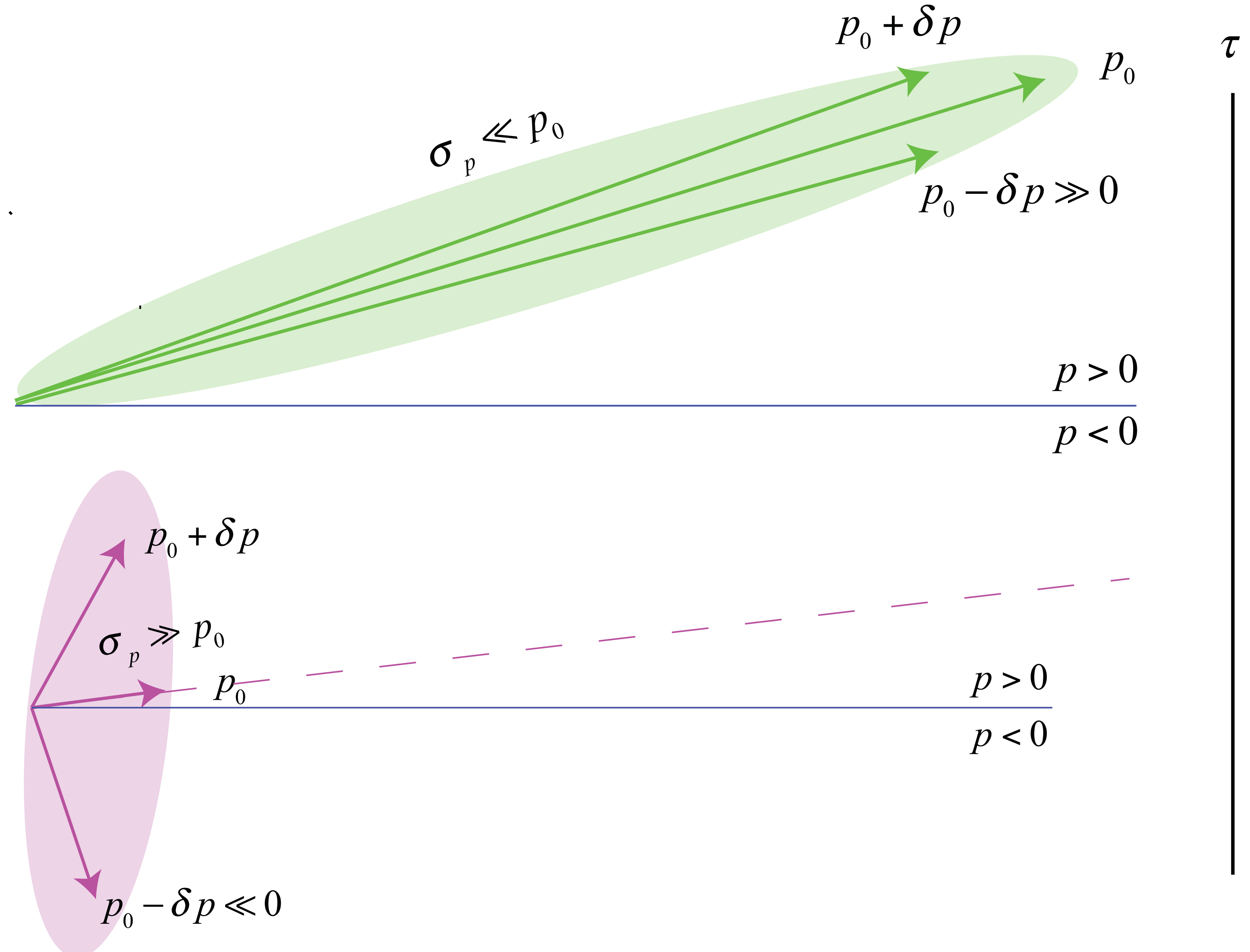
$$f_\tau^{(x)} \equiv 1 + \imath\frac{\tau}{m\sigma_x^2} \qquad \bar{x}_\tau \equiv x_0 + \frac{p}{m}\tau$$

$$\bar{\rho}_\tau(x) = \sqrt{\frac{1}{\pi\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}}\exp\left(-\frac{\left(x-\bar{x}_\tau\right)^2}{\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}\right)$$

$$\left(\Delta x\right)^2 \equiv \left\langle x_\tau^2\right\rangle - \bar{x}_\tau^2 = \frac{\sigma_x^2}{2}\left|1+\frac{\tau^2}{m^2\sigma_x^4}\right|$$

- Normalizable
- Satisfy free Schrödinger equation
- Back-of-envelope
- General

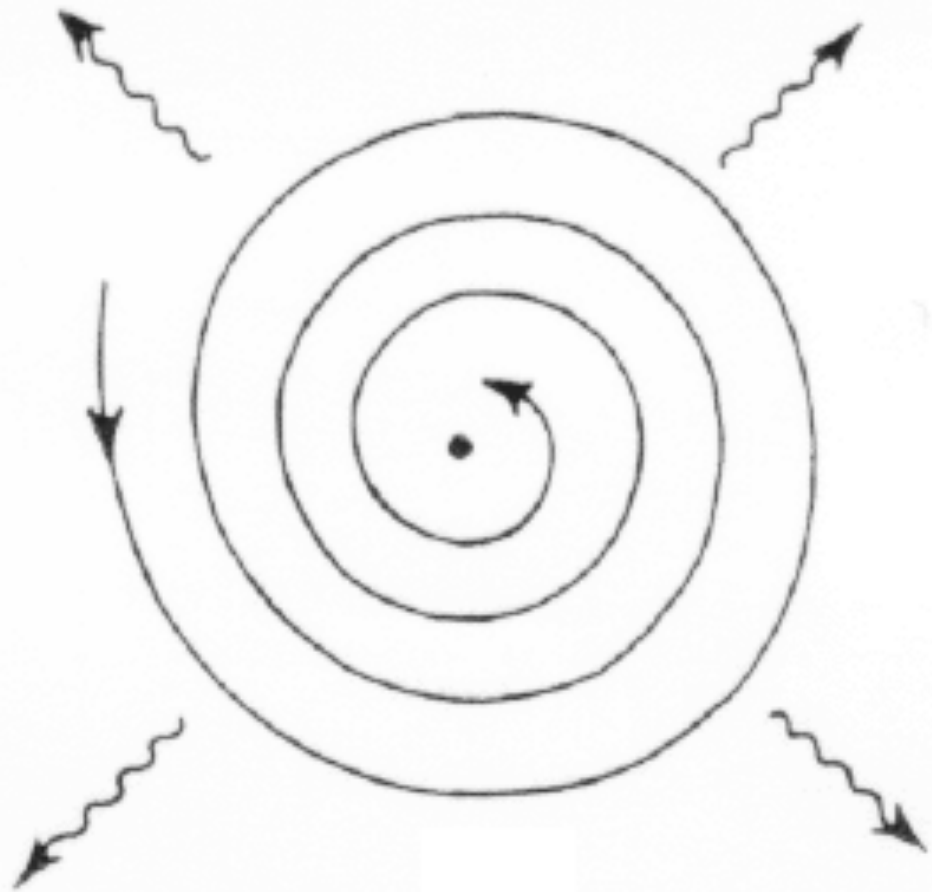
Bullet versus Wave



- Bullet works well
- But slow, wide wave-y GTF produces
- Nonsense

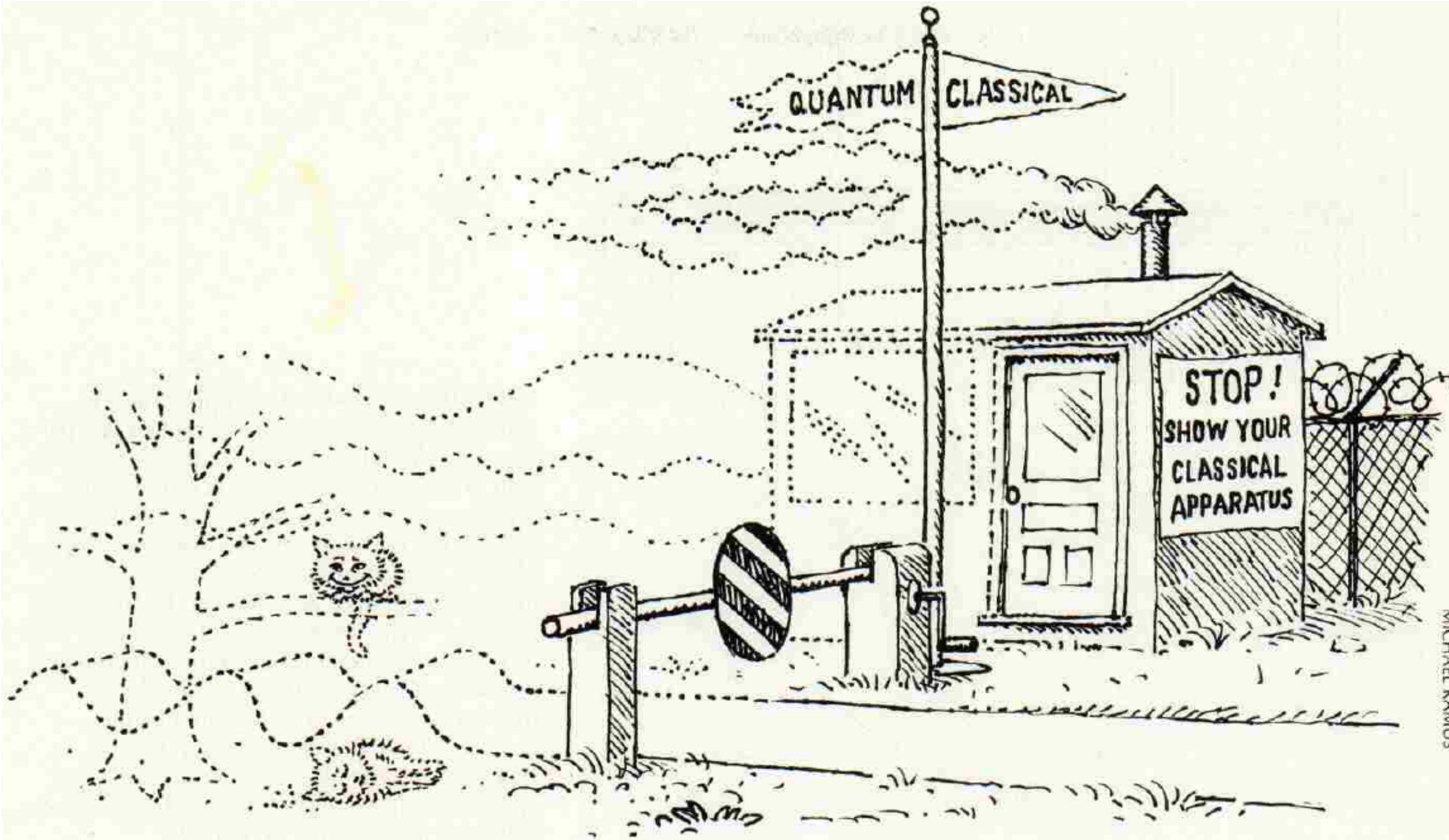
Life span of an atom

- accelerated electron radiates
- once its energy is gone, it spirals into the nucleus
- in about one hundred trillionth of a second!



According to classical physics, an electron in orbit around an atomic nucleus should emit electronmagnetic radiation (photons) continuously, because it is continually accelerating in a curved path. The resulting loss of energy implies that the electron should spiral into the nucleus in a very short time (i.e. atoms can not exist).

Checkpoint Copenhagen



Conservation of probability

Therefore by conservation of probability we have:

$$1 = \int_{-\infty}^{0} dx \psi_\tau^*(x)\psi_\tau(x) + \int_0^\tau d\tau' D_{\tau'}$$

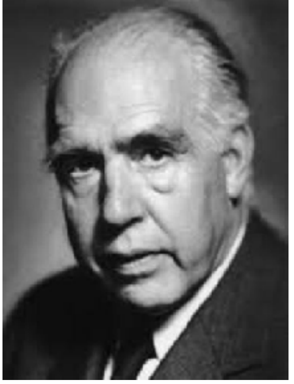
$$D_\tau = -\int_{-\infty}^{0} dx \frac{\partial \psi_\tau^*(x)}{\partial \tau}\psi_\tau(x) + \psi_\tau^*(x)\frac{\partial \psi_\tau(x)}{\partial \tau}$$

$$D_\tau = \frac{\imath}{2m}\int_{-\infty}^{0} dx \frac{\partial^2 \psi_\tau^*(x)}{\partial x^2}\psi_\tau(x) - \psi_\tau^*(x)\frac{\partial^2 \psi_\tau(x)}{\partial x^2}$$

$$D_\tau = \frac{\imath}{2m}\left(\frac{\partial \psi_\tau^*(x)}{\partial x}\psi_\tau(x) - \psi_\tau^*(x)\frac{\partial \psi_\tau(x)}{\partial x}\right)\Bigg|_{x=0}$$

$$J_\tau = \psi_\tau^* \frac{p}{2m}\psi_\tau - \psi_\tau \frac{p}{2m}\psi_\tau^*$$

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot \vec{J} = 0, \vec{J} \equiv \frac{1}{2m}\psi^* \vec{p}\psi - (\psi^* \vec{p})\psi$$

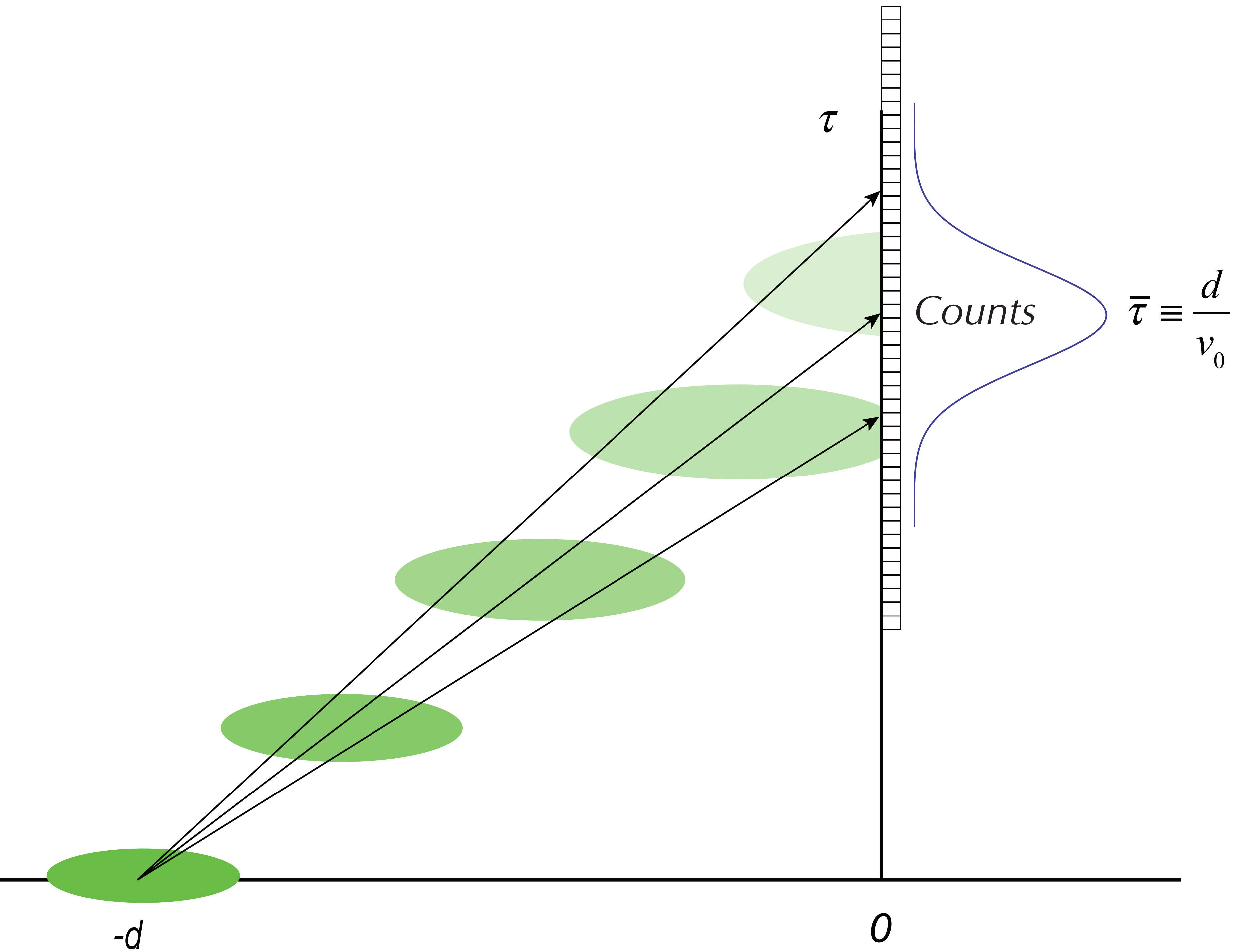


What makes QM QM?

"Indeed the finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails…the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality" — Bohr

- Quantum Zeno effect
- Anti-Zeno
- Backflow
- Partial detection
- Delayed detection
- …

GTF MEETS DETECTOR



- Detector only open for Δτ
- So doesn't interfere with itself
- Crude but reasonable

Results

$$D_\tau = \sqrt{\frac{1}{\pi\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}}\exp\left(-\frac{(d-v_0\tau)^2}{\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}\right)$$

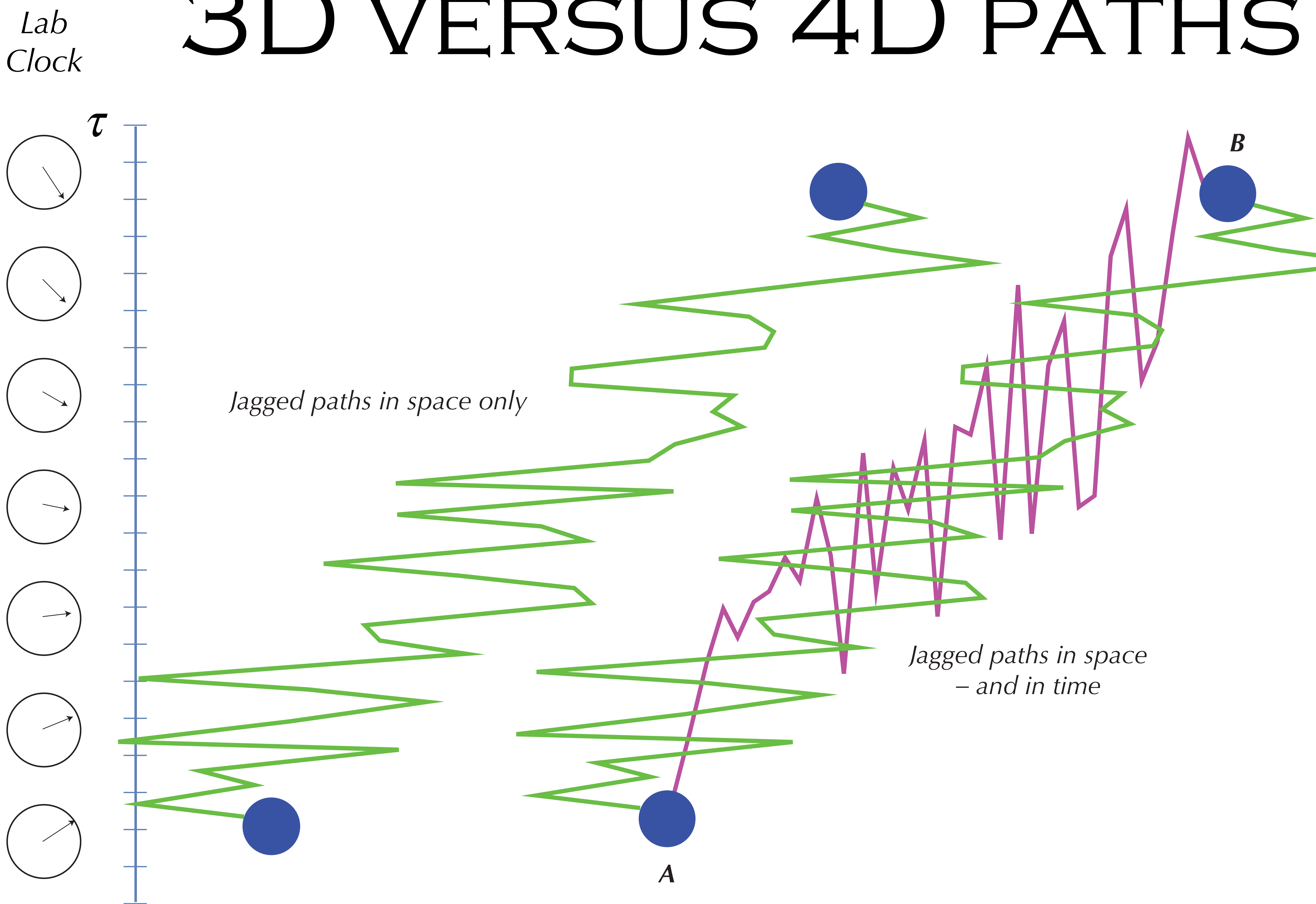
$$\tau = \bar{\tau} + \delta\tau, \bar{\tau} = \frac{d}{v_0}$$

$$D_\tau = \sqrt{\frac{1}{\pi\sigma_x^2\left(1+\frac{\bar{\tau}^2}{m^2\sigma_x^4}\right)}}\exp\left(-\frac{(\delta\tau)^2}{\sigma_x^2\left(1+\frac{\bar{\tau}^2}{m^2\sigma_x^4}\right)}\right)$$

$$\bar{\sigma}_\tau^2 = \frac{\bar{\tau}^2}{m^2 v_0^2 \sigma_x^2}$$

- Detection rate
- Steepest descents
- Dispersion in time
- Slower = wider

3D versus 4D paths



Derivation of Schrödinger equation

$$K_\tau(x'';x') = \lim_{N\to\infty} \int \mathcal{D}\pi e^{\imath \int_0^\tau d\tau' \mathcal{L}[\pi,\dot{\pi}]}$$

$$\mathcal{D}\pi \equiv \left(-\imath \frac{m^2}{4\pi^2\varepsilon^2}\right)^N \prod_{n=1}^{N-1} d^4 x_n$$

$$L(x^\mu, \dot{x}^\mu) = -\frac{1}{2} m \dot{x}^\mu \dot{x}_\mu - q\dot{x}^\mu A_\mu(x) - \frac{m}{2}$$

$$\imath \frac{\partial \psi_\tau}{\partial \tau} = -\frac{1}{2m}\left(\left(p_\mu - qA_\mu\right)\left(p^\mu - qA^\mu\right) - m^2\right)\psi_\tau$$

$$\imath \frac{\partial}{\partial \tau}\psi_\tau = -\frac{E^2 - p^2 - m^2}{2m}\psi_\tau$$

$$\imath \frac{\partial}{\partial \tau}\psi_\tau(t,\vec{x}) = \frac{1}{2m}\left(\frac{\partial^2}{\partial t^2} - \nabla^2 - m^2\right)\psi_\tau(t,\vec{x})$$

4D Paths

3D Paths

$$K_\tau(x'';x') = \lim_{N\to\infty} \int \mathcal{D}\pi e^{\imath \int_0^\tau d\tau' \mathcal{L}[\pi,\dot{\pi}]}$$

$$\mathcal{D}\pi \equiv \left(-\imath \frac{m^2}{4\pi^2\varepsilon^2}\right)^N \prod_{n=1}^{N-1} d^4 x_n$$

$$\mathcal{D}\pi \equiv \left(\sqrt{\frac{\imath m}{2\pi\varepsilon}}^3\right)^N \prod_{n=1}^{N-1} d^3 x_n$$

$$L(x^\mu,\dot{x}^\mu) = -\frac{1}{2} m \dot{x}^\mu \dot{x}_\mu - q \dot{x}^\mu A_\mu(x) - \frac{m}{2}$$

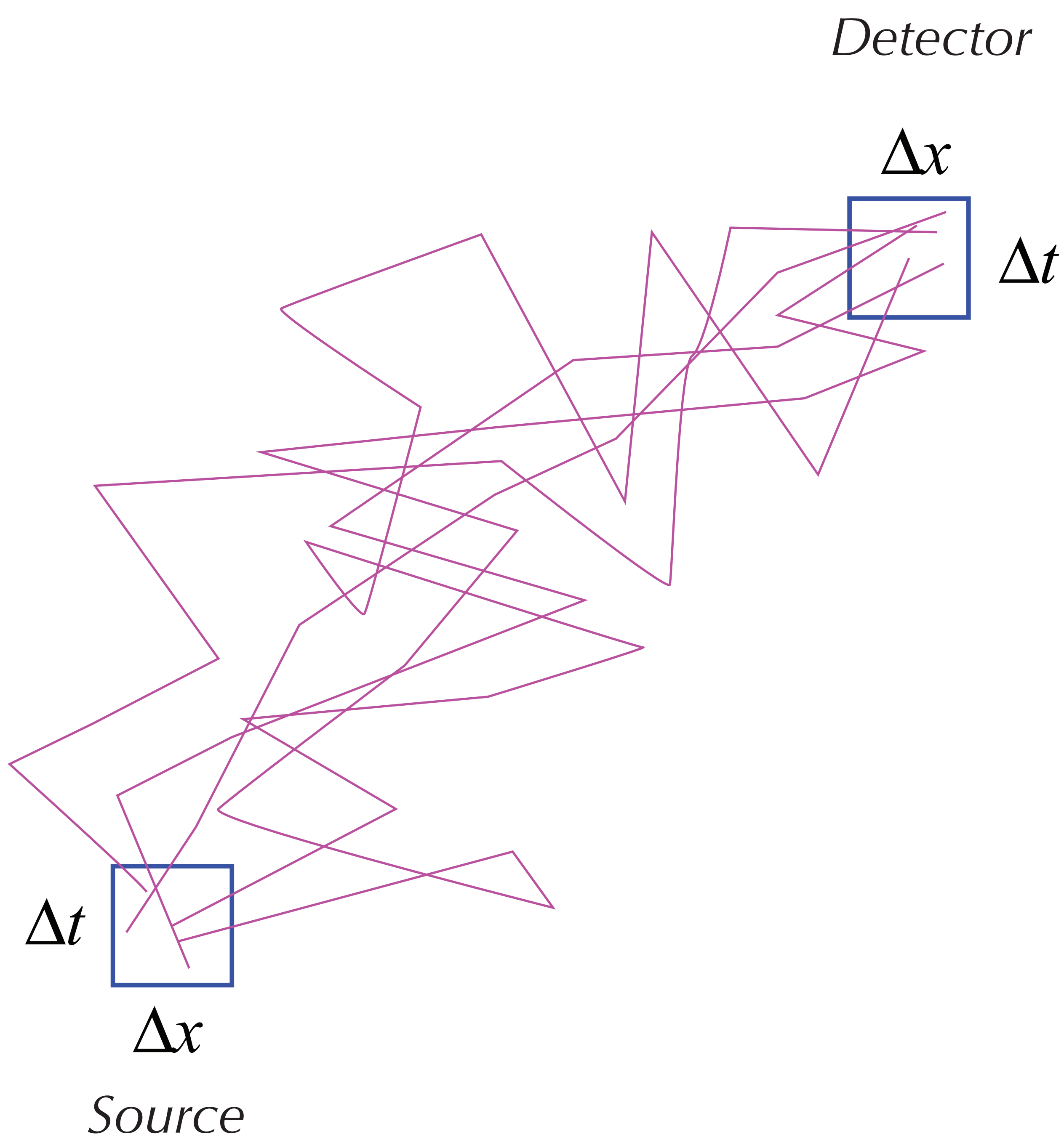
$$\tau = \langle t \rangle$$

$$\imath \frac{\partial}{\partial\tau} \psi_\tau(t,\vec{x}) = \frac{1}{2m}\left(\frac{\partial^2}{\partial t^2} - \nabla^2 - m^2\right)\psi_\tau(t,\vec{x})$$

$$0 = \left(\frac{\partial^2}{\partial\tau^2} - \nabla^2 - m^2\right)\psi_\tau(t,\vec{x})$$

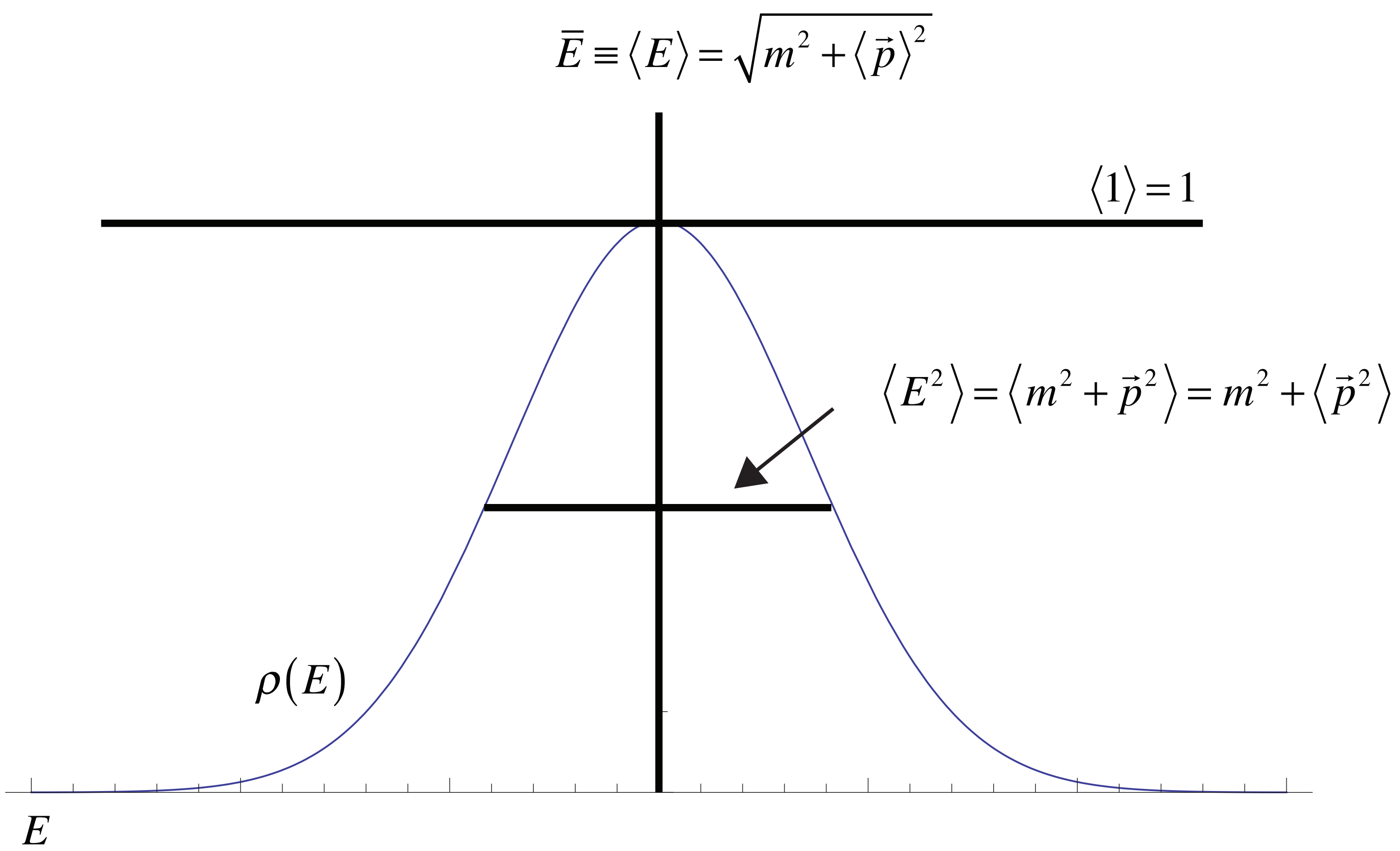
$$(\Delta t)^2 \equiv \langle t^2 - \tau^2 \rangle \sim 10^{-18}\,\text{sec}$$

Rules for detection



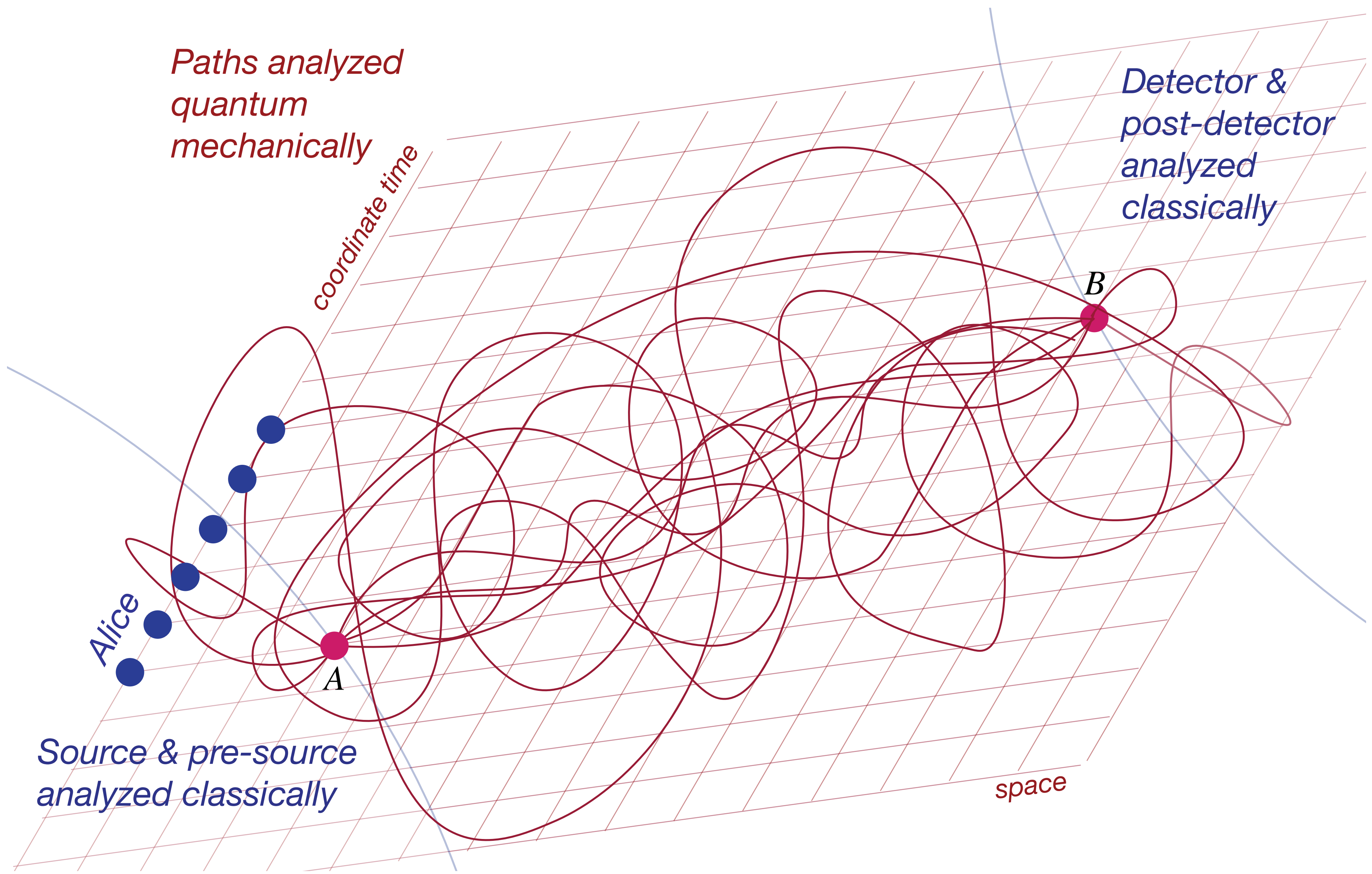
- Rules for detection must be the same in time as in space
- We are mapping from the coordinate in the path to the coordinate in detector
- Paths are tied at ends but free elsewhere

Initial Wave Function?



- We have constraints on norm, average, & dispersion
- Get maximum entropy solution using Lagrange multipliers

Clock time as average over the rest of the universe



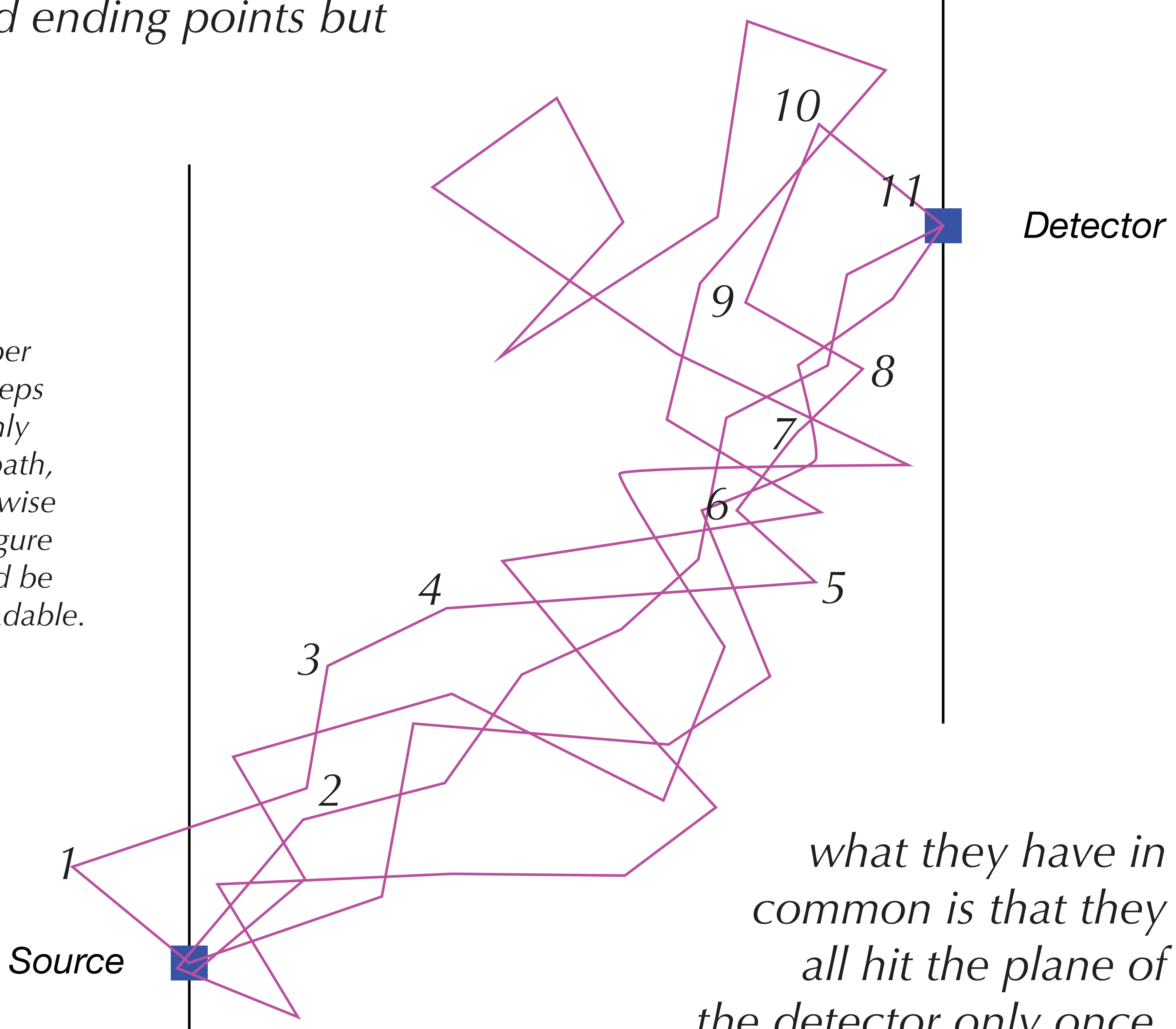
$$\tau \equiv \left\langle \psi_{rest-of-universe} \middle| t \middle| \psi_{rest-of-universe} \right\rangle$$

Quantum Pointillism

Convolutions over clock time

Different paths can take a wildly different number of steps to go between the same starting and ending points but

We number the steps for only one path, otherwise the figure would be unreadable.



what they have in common is that they all hit the plane of the detector only once.

Definite but small effects

$$\sigma_\tau^2 = \bar{\sigma}_\tau^2 + \tilde{\sigma}_\tau^2 = \frac{\bar{\tau}^2}{m^2 v_0^2 \sigma_x^2} + \frac{\bar{\tau}^2}{m^2 \sigma_t^2}$$

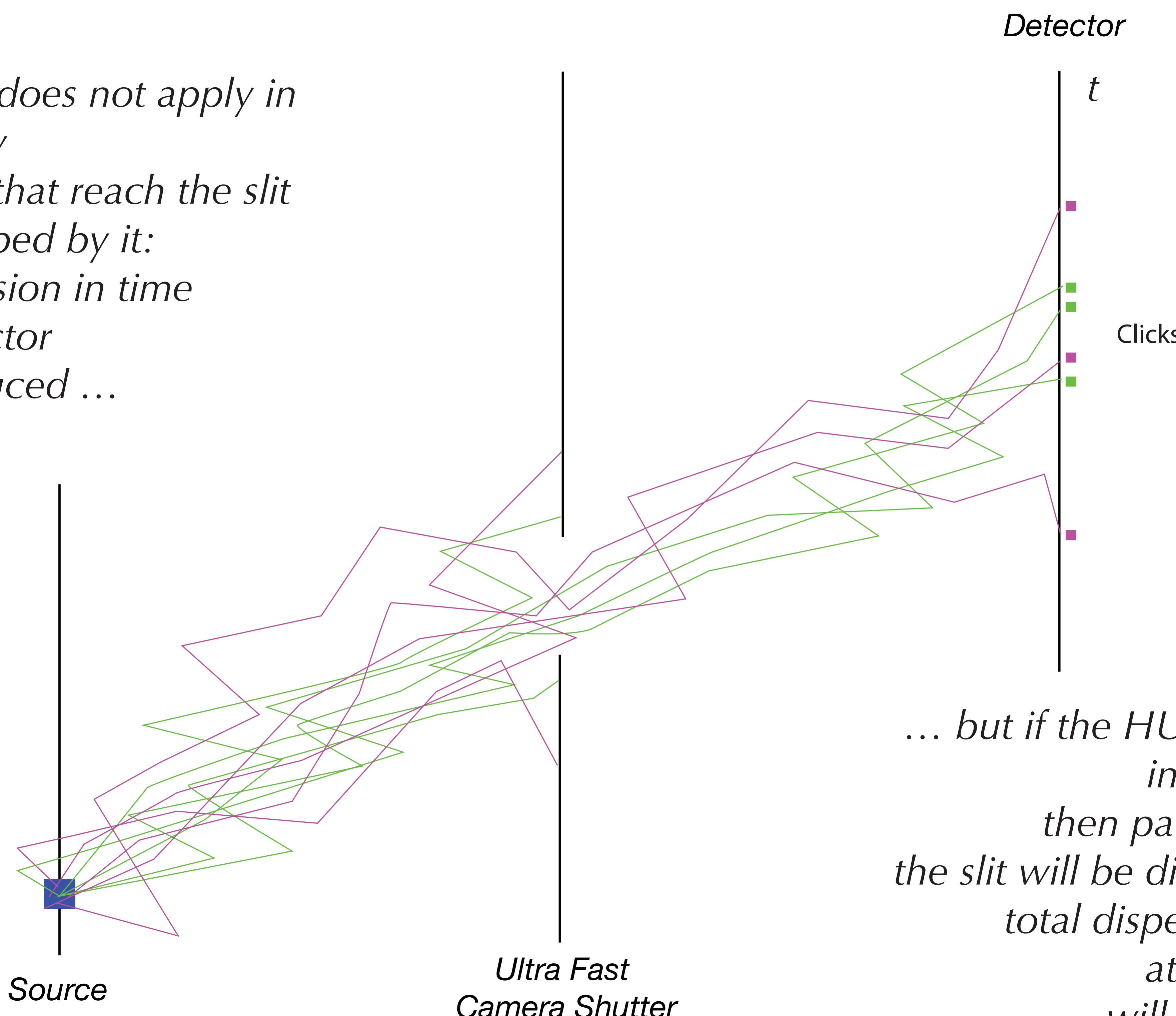
$$\sigma_t^2 \sim \sigma_x^2$$

$$\frac{1}{v_0} \gg 1 \Rightarrow \bar{\sigma}_\tau^2 \gg \tilde{\sigma}_\tau^2$$

- Small definite effects
- Present everywhere
- For non-relativistic objects, additional dispersion small

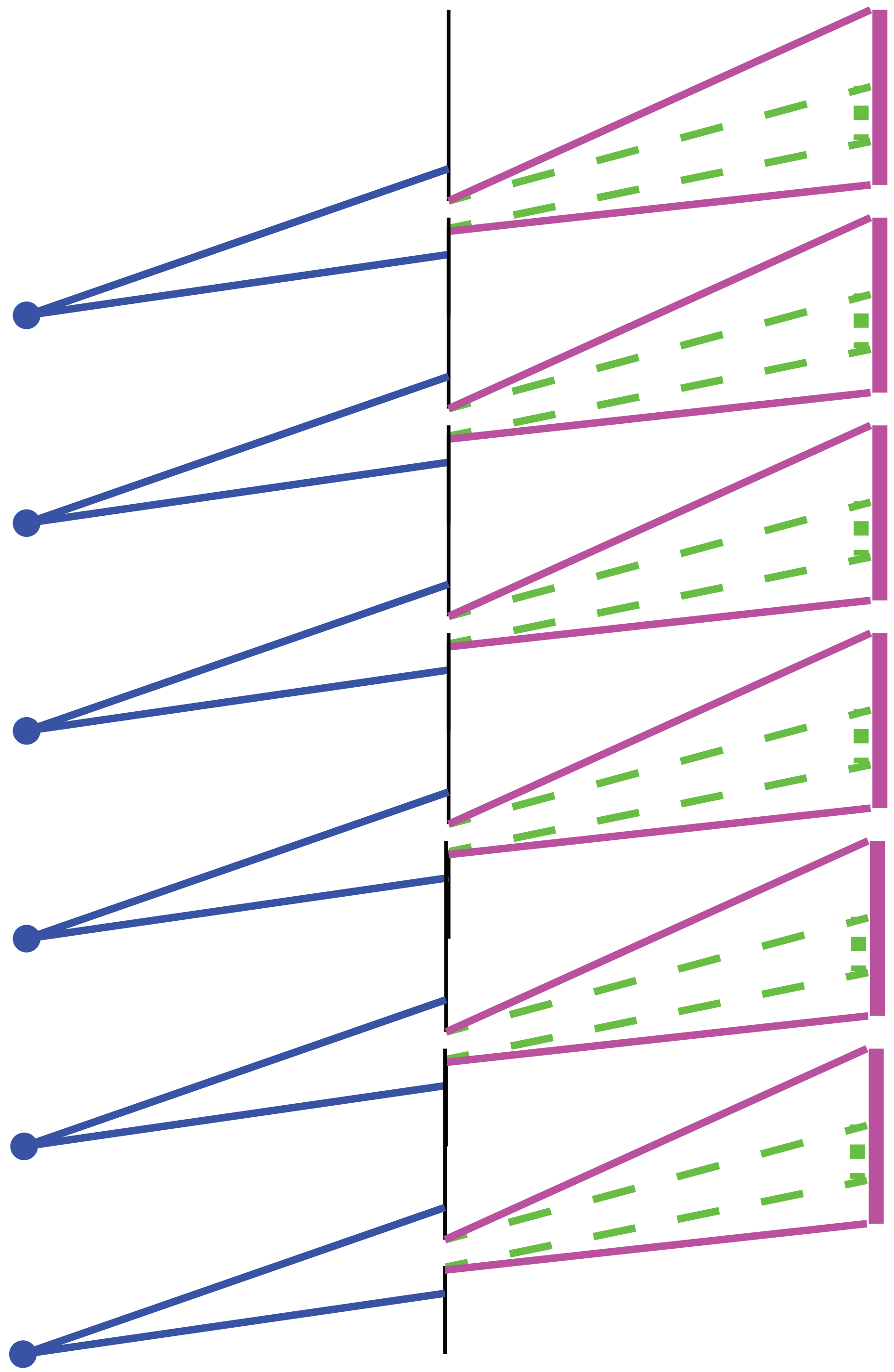
Ultra fast camera

If the HUP does not apply in time/energy
then paths that reach the slit
will be clipped by it:
total dispersion in time
at the detector
will be reduced …



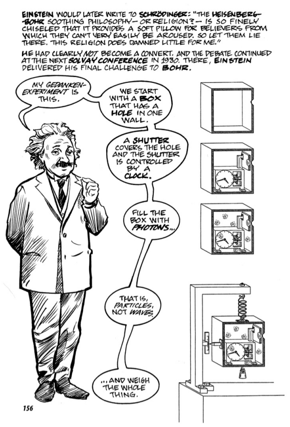
… but if the HUP does apply
in time/energy
then paths that reach
the slit will be diffracted by it:
total dispersion in time
at the detector
will be increased.

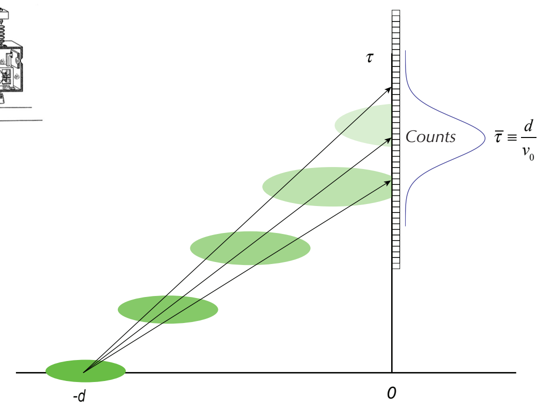
Technically challenging experiments

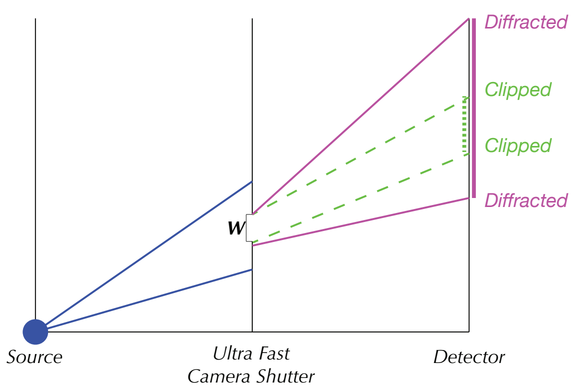


- Have to calculate time on a per particle basis (strobed?)
- Have to model the detector (& in time as well as space)
- Do not look like easy experiments

$\Delta E \Delta t \geq 1?$



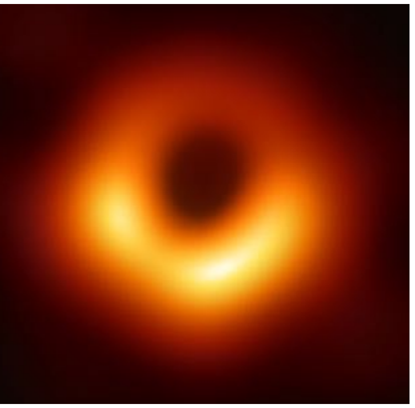


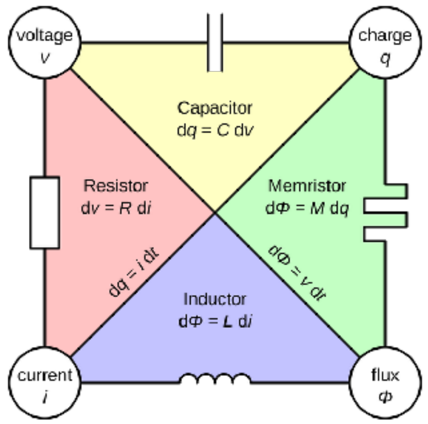


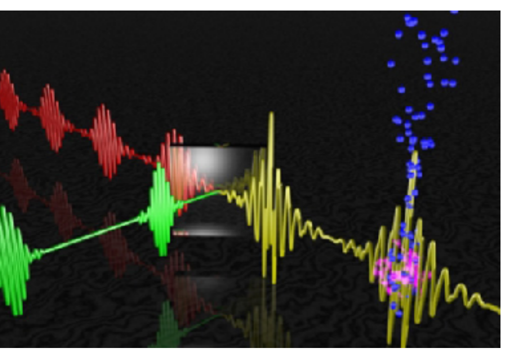
Standard error (10^{-18} s)
0.99
0.85
1.01
1.60

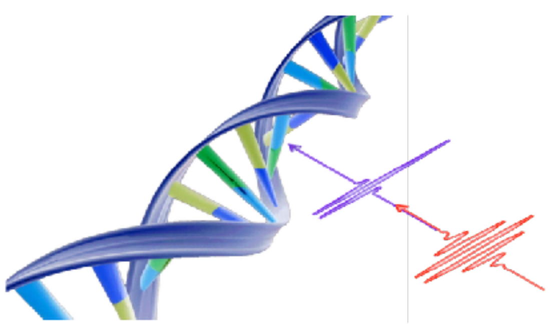
- More likely than not (from symmetry)
- Present everywhere, but small
- Present large, but in only specialized experiments
- Falsifiable — just not easy

$$(\Delta t)^2 \equiv \langle t^2 - \tau^2 \rangle \sim \text{attoseconds?}$$









- Quantum gravity without costs & risks of black holes
- New communication channels, new circuit elements, as memristors
- Attosecond chemistry; attoseconds are the time scale for electron/molecule interactions
- Attosecond biology; better understanding of DNA, protein formation, …
- Let's take a flyer on the future; you never know where the lightning will strike!