Quantum Properties and Gravitational Field of a Proper Time Oscillator

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June 2020

Abstract

We show that the basic properties of a spin-zero quantum field (e.g. Klein-Gordon equation, Schrdingers equation, probability density, second quantization, etc.) can emerge from a system with vibrations in space and time. The internal time of this system can be represented by a self-adjoint operator. The spectrum of this operator is unbounded and not restricted by Pauli's theorem. Also, the particle observed has oscillation in proper time. By neglecting all quantum effects and assuming the particle as a classical object that can remain stationary in space, we show that the proper time oscillator can mimic a point mass at rest in general relativity. The spacetime outside this proper time oscillator is static and satisfies the Schwarzschild solution. To measure the temporal oscillation, neutrino can be an interesting candidate because of its extremely light weight.

References

This presentation summarizes the following papers:

[1] H. Y. Yau, Schwarzschild field of a proper time oscillator, Symmetry 12(2), 312 (2020).
[2] H. Y. Yau, Self-adjoint time operator in a quantum field, Int, J. Quant. Info. 1941016 (2020)
[3] H. Y. Yau, Thin shell with fictitious oscillations, in Spacetime Physics 1907 2017, Chapter 6 (Minkowski Institute Press, Montreal, 2019)
[4] H. Y. Yau, Probabilistic Time and Space Symmetry in a Quantum Field, J. Phys.: Conf. Ser. 1194, 012116 (2019)
[5] H. Y. Yau, Temporal vibrations in a quantized field, in Quantum Foundations, Probability and Information, 269 (Springer, Verlag, 2018)
[6] H. Y. Yau, Probabilistic nature of a field with time as a dynamical variable, Lect. Notes Comp. Sci. 10106, 33 (2016)
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Key Results

- Spacetime around an infinitesimal thin shell with fictitious radial vibrations has the Schwarzschild field solution
- A proper time oscillator is the generator of the fictitious radial vibrations
- Matter field has temporal vibrations
- Reconcile basic properties of a non-interacting spin-zero matter wave
- Internal time of system as self-adjoint operator
- A $1/2$ spin particle as a rotating particle with oscillation in time

Past Examples Schwarzschild Metric without General Relativity

- Lenz W, unpublished work cited by Sommerfeld A (1944)
- Schiff L I, On experimental tests of the general theory of relativity, Am. J. Phys. 28 340-343 (1960)
- Laschkarew W, Zur Theorie der Gravitation Z.Physik, 35 473-476 (1926)
- Harwit M, Astrophysical Concepts (Wiley, New York, 1973).
- Rowlands P, A simple approach to the experimental consequences of general relativity, Physics Education 32, 49 (1997).
- Czerniawski J, The possibility of a simple derivation of the Schwarzschild metric, *Preprint* gr-qc/0611104.
- Cuzinatto R et.al, Schwarzschild and de Sitter solution from the argument by Lenz and Sommerfiled, Am. J. Phys. 79, 662 (2011).

$$ds^2 = [1 - \underline{v}^2]dt^2 - [1 - \underline{v}^2]^{-1}dx^2 - dy^2 - dz^2. \tag{1}$$

Problems and Solutions

- Counterexamples: Gruber R P, Price R H, Matthews S M, Cordwell W R and Wagner L F 1988 The impossibility of a simple derivation of the Schwarzschild metric Am. J. Phys. 56 265-269
- Counterexamples: Rindler W 1968 Counterexample to the Lenz-Schiff argument Am. J. Phys. 36 540-544 "
- Kassner; Impossible only with SR, EP and NL
- "It is the spatial-distortion aspect of gravity that ensures that too simple a derivation of the Schwarzschild metric must fail"
- Instead of trying to reconcile the Schwarzschild spacetime geometry, we apply the idea to a thin shell with fictitious velocity

$$ds^2 = [1 - \underline{v}^2]dt^2 - [1 - \underline{v}^2]^{-1}dx^2 - dy^2 - dz^2. \tag{2}$$

Thin Shell with Fictitious Radial Oscillations

Consider an infinitesimally thin spherical shell Σ with radius $\breve{r}(> 2m)$. Relative to this shell, there are radial oscillations, i.e.

$$\mathring{t}_f(t, \breve{r}) = t, \tag{3}$$

$$\mathring{r}_f(t, \breve{r}) = \breve{r} + \breve{\Re} \cos(\omega_0 t), \tag{4}$$

The instantaneous radial velocity is,

$$\mathring{v}_f(t, \breve{r}) = \frac{\partial}{\partial t} \mathring{r}_f(t, \breve{r}) = -\breve{\Re}\omega_0 \sin(\omega_0 t) < 1, \tag{5}$$

Thin Shell with Fictitious Radial Oscillations

- Clock of a fictitious observer $\underline{O}$ oscillating about $r = \check{r}$ synchronizes with the clock of an observer O at spatial infinity.
- In its fictitious frame, $\underline{O}$ is an inertial observer.
- An observer $\check{O}$ stationary at $r = \check{r}$ has a fictitious displacement $\underline{r}_f$ and instantaneous velocity $\underline{v}_f$ relative to $\underline{O}$,

$$\underline{r}_f(t, \check{r}) = -\mathring{r}_f(t, \check{r}) + \check{r} = -\breve{\mathfrak{R}} \cos(\omega_0 t), \tag{6}$$

$$\underline{v}_f(t, \check{r}) = -\mathring{v}_f(t, \check{r}) = \breve{\mathfrak{R}} \omega_0 \sin(\omega_0 t). \tag{7}$$

- Although $\check{O}$ is stationary relative to O at spatial infinity, it is under the effects as if $\check{O}$ is oscillating in the fictitious frame of $\underline{O}$.

Effects at $t = t_m = \pi/(2\omega_0)$

- Obtain the spacetime geometrical properties of the thin shell Σ when there is only a fictitious velocity$|\underline{v}_f| < 1$.
- At $t = t_m = \pi/(2\omega_0)$, the fictitious displacement and instantaneous velocity are:

$$\underline{r}_f(t_m, \check{r}) = \underline{r}_{fm} = 0, \tag{8}$$

and

$$\underline{v}_f(t_m, \check{r}) = \underline{v}_{fm} = \check{\Re}\omega_0 < 1. \tag{9}$$

- $\check{O}$ is traveling with a velocity $\underline{v}_{fm}$ in the fictitious frame with no displacement relative to $\underline{O}$.
- Understand how the clocks and measuring rods carried by O and $\check{O}$ are related at the instant $t = t_m$.

Time and Spatial Measurements in $\check{O}$

- Consider two events in frame $\check{O}$ relate to the coordinate increments dt and dr for the same two events observed in frame O,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} \Upsilon^{t}{}_{\check{t}} & \Upsilon^{t}{}_{\check{r}} \\ \Upsilon^{r}{}_{\check{t}} & \Upsilon^{r}{}_{\check{r}} \end{bmatrix} \begin{bmatrix} d\check{t} \\ d\check{r} \end{bmatrix}. \tag{10}$$

- In the local frames, the basis vectors in the temporal and radial directions are orthogonal, i.e. $\mathbf{e}_t \cdot \mathbf{e}_r = 0$ and $\mathbf{e}_{\check{t}} \cdot \mathbf{e}_{\check{r}} = 0$.
- $\check{O}$ is stationary relative to the inertial frame O.
- $\mathbf{e}_{\check{t}} \parallel \mathbf{e}_t$, and $\mathbf{e}_{\check{r}} \parallel \mathbf{e}_r$.

$$\Upsilon^{t}{}_{\check{r}} = \Upsilon^{r}{}_{\check{t}} = 0. \tag{11}$$

Clocks in $\check{O}$ and O

- When $d\check{r} = 0$, $d\check{t}$ is a proper time measured by the clock carried by $\check{O}$.
- Lorentz transformed to the fictitious frame of $\underline{O}$,

$$d\underline{t} = \gamma d\check{t}, \tag{12}$$

$$d\underline{r} = \underline{\gamma \underline{v}_{fm}} d\check{t}, \tag{13}$$

- In the fictitious frame, $\check{O}$ travels a distance $d\underline{r}$ over a time $d\underline{t}$.
- Clocks of O and $\underline{O}$ are synchronized.
- O shall measure the same time as $\underline{O}$,

$$dt = d\underline{t} = \gamma d\check{t}. \tag{14}$$

- O is physically stationary relative to $\check{O}$,

$$dr = 0. \tag{15}$$

- Effect slows down the clock of $\check{O}$ but without relative movement between O and $\check{O}$.

$$\Upsilon^{t}{}_{\check{t}} = \gamma = [1 - (\underline{v}_{fm})^2]^{-1/2} = (1 - \check{\Re}^2 \omega_0^2)^{-1/2}. \tag{16}$$

Measuring Rods in $\check{O}$ and O

- Consider a measuring rod $d\check{r}$ carried by $\check{O}$, expressed as two events measured at the endpoints of the rod simultaneously, $d\check{t} = 0$.
- Lorentz transform to the fictitious frame $\underline{O}$,

$$d\underline{t} = \underline{\gamma \underline{v}_{fm} d\check{r}}, \tag{17}$$

$$d\underline{r} = \gamma d\check{r}. \tag{18}$$

- From viewpoint of $\underline{O}$, rod carried by $\check{O}$ is moving at a velocity $\underline{v}_{fm}$.
- Moving length $d\underline{l}$ of the rod,

$$d\underline{l} = d\underline{r} - \underline{\underline{v}_{fm} d\underline{t}} = \gamma^{-1} d\check{r}. \tag{19}$$

- As inertial observers with their clocks synchronized, O measures the same length of the rod as $\underline{O}$,

$$dr = d\underline{l} = \gamma^{-1} d\check{r}. \tag{20}$$

Measuring Rods in $\breve{O}$ and O

- A rod carried by $\breve{O}$ is stationary relative to O.
- The effects shorten the rod observed in frame O but there is no relative movement between O and $\breve{O}$.
- Length of rod in frame O measured simultaneously at the endpoints, $dt = 0$.

$$\Upsilon^{r}{}_{\breve{r}} = \gamma^{-1} = [1 - (\underline{v}_{fm})^2]^{1/2} = (1 - \breve{\Re}^2 \omega_0^2)^{1/2}. \qquad (21)$$

Symmetry under Time Translation

- What is the effect of the fictitious displacement?
- The simple harmonic oscillating system has a symmetry under time translation.
- Effects of the fictitious oscillations on $\breve{O}$ shall be constant over time.
- Define a constant,

$$\breve{I} = \omega_0^2(\underline{r}_f)^2 + (\underline{v}_f)^2 = \breve{\Re}^2\omega_0^2, \tag{22}$$

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} (1-\breve{I})^{-1/2} & 0 \\ 0 & (1-\breve{I})^{1/2} \end{bmatrix} \begin{bmatrix} d\breve{t} \\ d\breve{r} \end{bmatrix}, \tag{23}$$

Line Element at $r = \check{r}$

- Relate the basis vectors in frame O and $\check{O}$,

$$\mathbf{e}_{\check{t}} = \mathbf{e}_t(1 - \check{I})^{1/2}, \tag{24}$$

$$\mathbf{e}_{\check{r}} = \mathbf{e}_r(1 - \check{I})^{-1/2}. \tag{25}$$

- Line element at $r = \check{r}$,

$$ds^2 = g_{tt}(\check{r})dt^2 + 2g_{tr}(\check{r})dtdr + g_{rr}(\check{r})dr^2 - \check{r}^2 d\Omega^2, \tag{26}$$

- Same coordinate system adopted for the conventional Schwarzschild field.

- Metrics at O and $\check{O}$ are different.

$$g_{tt}(\check{r}) = \mathbf{e}_{\check{t}} \cdot \mathbf{e}_{\check{t}} = (1 - \check{l})\mathbf{e}_t \cdot \mathbf{e}_t = 1 - \check{l}, \tag{27}$$

$$g_{rr}(\check{r}) = \mathbf{e}_{\check{r}} \cdot \mathbf{e}_{\check{r}} = (1 - \check{l})^{-1}\mathbf{e}_r \cdot \mathbf{e}_r = -(1 - \check{l})^{-1}, \tag{28}$$

$$g_{tr}(\check{r}) = g_{rt}(\check{r}) = \mathbf{e}_{\check{t}} \cdot \mathbf{e}_{\check{r}} = \mathbf{e}_t \cdot \mathbf{e}_r = 0, \tag{29}$$

- Line element at $r = \check{r}$ is,

$$ds^2 = [1 - \check{l}]dt^2 - [1 - \check{l}]^{-1}dr^2 - \check{r}^2 d\Omega^2. \tag{30}$$

- System also invariant under time reflection symmetry ($t \rightarrow -t$).

Schwarzschild Field

Metric at $r = \breve{r}$ is line element of Schwarzschild if

$$\breve{l} = \frac{2m}{\breve{r}}, \tag{31}$$

or

$$m = \frac{\breve{r}\breve{\Re}^2\omega_0^2}{2}. \tag{32}$$

The vacuum spacetime υ^+ outside this time-like hypersurface is the Schwarzschild spacetime,

$$ds^2 = [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]dt^2 - [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]^{-1}dr^2 - r^2 d\Omega^2. \tag{33}$$

Contraction of Thin Shell

- Time-like hypersurface Σ can be contracted per Birkhoffs theorem.
- As long as mass m of the shell is remaining constant, the metric and curvature of the external field will not be affected.
- The amplitude of the radial oscillation is, $\breve{\mathfrak{R}} = \sqrt{\frac{2}{\breve{r}\omega_0}}$.
- Spacetime curvature tensors derived are well defined as the shell is contracted until it reaches a radius $\breve{r} = \epsilon/2$.
- Shell becomes infinitely small but with $\breve{\mathfrak{R}} \to \infty$.
- Although fictitious instantaneous velocity on a shell inside event horizon can exceed the speed of light (i.e. $\underline{v}_{fm} > 1$ when $\breve{r} < 2m$), they are not physical vibrations of matter.
- As information about the geometrical properties of spacetime, there is no superluminal transfer of energy.
- As predicted by Birkhoffs theorem, the metric around this infinitely small shell is the Schwarzschild spacetime.

Next, consider a proper time oscillator

$$\mathring{t}_f(t, \mathbf{x}) = t - \frac{\Pi(\mathbf{x})\sin(\omega_0 t)}{\omega_0} = t + \mathring{\zeta}_t(t, \mathbf{x}), \tag{34}$$

where

$$\mathring{\zeta}_t(t, \mathbf{x}) = -\frac{\Pi(\mathbf{x})}{\omega_0}\sin(\omega_0 t), \tag{35}$$

and

$$\Pi(\mathbf{x}) = 0 \text{ if } |\mathbf{x}| \geq \epsilon/2, \tag{36}$$

$$\Pi(\mathbf{x}) = 1 \text{ if } |\mathbf{x}| < \epsilon/2. \tag{37}$$

$\Pi(\mathbf{x})$ is a pulse with width $\epsilon \to 0$.

Superposition of Temporal Oscillator

- $\mathring{\zeta}_t$ is a wave packet with infinitesimal width that can be decomposed into Fourier series of plane waves $\zeta_{t\mathbf{k}} = -iT_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$.
- Utilize $\zeta_{t\mathbf{k}}$ as functions for the Fourier decomposition of a classical wave.
- $\zeta_{t\mathbf{k}}$ is only the 0-component of a plane wave with 4-vector amplitude.
- The other component with vibrations in space, $\zeta_{\mathbf{xk}} = -i\mathbf{X}_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$
- Further define a plane wave,

$$\zeta_{\mathbf{k}} = \frac{T_0}{\omega_0}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \tag{38}$$

such that ζ_t and $\zeta_{\mathbf{x}}$ can be obtained from ζ:

$$\zeta_{t\mathbf{k}} = \partial_0\zeta = -iT_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \tag{39}$$

$$\zeta_{\mathbf{xk}} = -\nabla\zeta == -i\mathbf{X}_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{40}$$

Superposition of Temporal Oscillator

Similarly, we can define for a proper time oscillator,

$$\mathring{\zeta}(t, \mathbf{x}) = \frac{\Pi(\mathbf{x})}{\omega_0^2} \cos(\omega_0 t), \tag{41}$$

which can be decomposed. The superpositions are linear, previous results can be extended to obtain its vibrations in time and space $(\mathring{\zeta}_t, \mathring{\zeta}_{\mathbf{x}})$, i.e.

$$\mathring{\zeta}_t = \frac{\partial \mathring{\zeta}}{\partial t}, \tag{42}$$

$$\mathring{\zeta}_{\mathbf{x}} = -\nabla \mathring{\zeta}. \tag{43}$$

There are additional oscillations in space other than the oscillation in time.

Radial Oscillations

System is spherically symmetric; switch to a spherical coordinate system,

$$\mathring{r}_f(t,r) = r + \mathring{\zeta}_r(t,r), \tag{44}$$

where

$$\mathring{\zeta}_r(t,r) = -\frac{\partial \mathring{\zeta}}{\partial r} = -\frac{\Pi'(r)}{\omega_0^2}\cos(\omega_0 t). \tag{45}$$

$\Pi'(r)$ denotes the derivative of $\Pi(r)$ with respect to r, such that

$$\Pi'(r) = 0 \text{ if } r \neq \epsilon/2, \tag{46}$$

$$\Pi'(r) = -\infty \text{ if } r = \epsilon/2. \tag{47}$$

- There are oscillations in the radial direction about $r = \epsilon/2$.

Rest Mass System

Summarize our results:

At $r = 0$,

$$\mathring{t}_f(t, 0) = t - \frac{\sin(\omega_0 t)}{\omega_0}, \tag{48}$$

$$\mathring{r}_f(t, 0) = 0. \tag{49}$$

At $r = \epsilon/2$,

$$\mathring{t}_f(t, \epsilon/2) = t, \tag{50}$$

$$\mathring{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t) \text{ with } \Re_\infty \to \infty. \tag{51}$$

Time Translation Symmetry

- The system has two oscillating components: the proper time oscillator at $r = 0$ and the radial oscillations about $r = \epsilon/2$.
- Total energies of simple harmonic oscillating systems are conserved.
- System as a whole has a symmetry under time translation by Noether's theorem.
- Internal energy E is the summation of two parts.
- Analogous to the 'potential' and 'kinetic' energy components of a classical harmonic oscillator.
- $E = mc^2$ looks like the energy of an oscillator. c is the rate of time.

Radial Oscillations not Vibration of Matter through Space

- Instantaneous velocity of the radial oscillation is,

$$\mathring{v}_f(t, \epsilon/2) = \frac{\partial}{\partial t}\mathring{r}_f(t, \epsilon/2) = -\Re_\infty \omega_0 \sin(\omega_0 t), \tag{52}$$

- Matter cannot have superluminal motion.
- Radial oscillation cannot be interpreted as vibration carrying an observer through space.
- Study the effects of these radial oscillation on an observer that is stationary at $r = \epsilon/2$; as a geometrical property of spacetime.

Fictitious Radial Oscillations

- An observer O stationary at spatial infinity is an inertial observer which is used as reference.
- In a Minkowski spacetime, the clock of a stationary observer at any location shall be synchronized.
- It is the clock of a fictitious observer $\underline{O}$ oscillating about $r = \epsilon/2$ that synchronize with the clock of O.

$$\mathring{t}_f(t, \epsilon/2) = t, \tag{53}$$

$$\mathring{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t) \text{ with } \Re_\infty \to \infty. \tag{54}$$

- An observer O_+ on the shell with radius $r = \epsilon/2$ will have an oscillation $\underline{r}_f$ relative to the fictitious inertial observer $\underline{O}$, i.e.

$$\underline{r}_f(t, \epsilon/2) = -\mathring{r}_f(t, \epsilon/2) + \epsilon/2 = -\Re_\infty \cos(\omega_0 t). \tag{55}$$

- This infinitesimal thin shell is the same as described earlier.
- Spacetime around the proper time oscillator is the Schwarzschild field.

Matter Field with Oscillations in Time

- Consider background coordinates $(t, \mathbf{x})$ in an inertial frame O
- Time in this background is the 'external time'
- Analogous to classical amplitude $\mathbf{X}$, define amplitude T as maximum difference between time of matter inside the wave, t_f, and the external time, t
- An inertial observer outside see the matter's clock vibrates with time
- Matter's internal clock running at a varying rate
- 'Internal time' t_f is an intrinsic property of matter
- $(T, \mathbf{X})$ is a 4-vector, where $T^2 = T_0^2 + |\mathbf{X}|^2$

Oscillations in Space and Time

The vibrations in space and time can be written as

$$\mathbf{x}_f = \mathbf{x} + \mathbf{X}\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = \mathbf{x} + \mathbf{x}_d = \mathbf{x} + \mathrm{Re}(\zeta_{\mathbf{x}}), \tag{56}$$

$$t_f = t + T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = t + t_d = t + \mathrm{Re}(\zeta_t), \tag{57}$$

where

$$\mathbf{x}_d = \mathrm{Re}(\zeta_{\mathbf{x}}) = \mathbf{X}\sin(\mathbf{k}\cdot\mathbf{x} - \omega t). \tag{58}$$

$$t_d = \mathrm{Re}(\zeta_t) = T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t), \tag{59}$$

- External time t used as reference measuring temporal vibrations
- Temporal vibrations as additional degrees of freedom
- Analyze only these two equations give us the quantum field

Real Scalar Field

Further define a plane wave,

$$\zeta = \frac{T_0}{\omega_0} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{60}$$

such that ζ_t and $\zeta_{\mathbf{x}}$ can be obtained from ζ as:

$$\zeta_t = \partial_0 \zeta, \tag{61}$$

$$\zeta_{\mathbf{x}} = -\nabla \zeta. \tag{62}$$

Vibrations of matter in space and time can be described by ζ.

Hamiltonian Density

- Consider a system in a volume V that can have multiple particles with mass m
- Impose periodic boundary conditions at the box walls

Equations of motion:

$$\partial_u \partial^u \zeta + \omega_0^2 \zeta = 0, \tag{63}$$

$$\partial_u \partial^u \zeta^* + \omega_0^2 \zeta^* = 0. \tag{64}$$

Classical field so far. The corresponding Hamiltonian density

$$\mathcal{H} = K[(\partial_0 \zeta^*)(\partial_0 \zeta) + (\nabla \zeta^*) \cdot (\nabla \zeta) + \omega_0^2 \zeta^* \zeta]. \tag{65}$$

Make the ansätz

$$K = \frac{m\omega_0^2}{2V}. \tag{66}$$

Analyze Hamiltonian Density Equation

Look at each term on the right hand side (RHS) of Hamiltonian density equation.

$$\mathcal{H}_1 = \left(\frac{m\omega_0^2}{2V}\right) T^* T, \tag{67}$$

$m\omega_0^2/2$ is an usual term of a harmonic oscillator with mass m except the vibration is in time.

$$\mathcal{H}_2 = \left(\frac{m\omega_0^2}{2V}\right) \mathbf{X}^* \cdot \mathbf{X}, \tag{68}$$

has the familiar form of harmonic oscillation in space.

$$\mathcal{H}_3 = \left(\frac{m\omega_0^2}{2V}\right) T_0^* T_0. \tag{69}$$

After combining,

$$\mathcal{H} = \left(\frac{m\omega_0^2}{V}\right) T^* T. \tag{70}$$

Internal Energy with Oscillation in Time

Consider the simple plane waves

$$\zeta_0 = \frac{T_0}{\omega_0} e^{-i\omega_0 t}. \tag{71}$$

Matter inside has vibrations in proper time only, i.e. $|\mathbf{k}| = 0$ and $\mathbf{x}_f = \mathbf{x}$.

$$\mathcal{H}_0 = (\frac{m\omega_0^2}{V}) T_0^* T_0. \tag{72}$$

- Inside the system, energy $E = m\omega_0^2 T_0^* T_0$
- Proper time is an intrinsic property of matter
- Only consider mass m without various charges or force fields.
- Consider this energy as the internal energy of mass.

Proper Time Oscillator

The energy E for the vibration of matter in proper time is necessary on shell if it is the internal energy of mass. For a single particle system, we have

$$E = m\omega_0^2 T_0^* T_0 = m, \tag{73}$$

or

$$\omega_0^2 T_0^* T_0 = 1. \tag{74}$$

- Suggest a possibility that a point mass m can have oscillation in proper time with amplitude $|\tilde{T}_0| = 1/\omega_0$.
- $E = mc^2$ looks like the energy of an oscillator. c is the rate of time.

Time inside the Oscillator

The internal time is:

$$\mathring{t}_f^+(t) = t - \frac{\sin(\omega_0 t)}{\omega_0}. \tag{75}$$

This point mass is stationary in space. The internal time rate is

$$\frac{\partial \mathring{t}_f^+}{\partial t} = 1 - \cos(\omega_0 t). \tag{76}$$

- The average of this time rate is 1
- Bounded between 0 and 2 which is positive moves only in the forward direction
- Particle will be observed traveling along a near time-like geodesic if our measurement is not sensitive enough.

Many Particle Field

The condition that mass is on shell imposes a constraint. We can extend this concept to a many particle system.

$$\omega_0^2 T_0^* T_0 = n, \tag{77}$$

which is a Lorentz invariant. The energy in a plane wave $\varphi_0^{\pm}$ with vibrations in proper time is quantized with $n = 0, 1, 2, ...$ For our analysis, we define another plane wave $\varphi_n^{\pm}$ which is normalized in volume V when $n = 1$,

$$\varphi_n^{\pm} = \gamma^{-1/2} \varphi^{\pm}, \tag{78}$$

where $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$. The Hamiltonian density is

$$H_n^{\pm} = \gamma H_0^{\pm} = \frac{n\omega}{V}. \tag{79}$$

The energy is quantized with n particles in a volume V.

Superposition of Wave

Obtain a real scalar field by superposition of plane waves,

$$\begin{aligned}\varphi(x) &= \sum_{\mathbf{k}} \varphi^{+}_{n\mathbf{k}}(x) + \varphi^{-}_{n\mathbf{k}}(x) \\ &= \sum_{\mathbf{k}} (2V\omega)^{-1/2} (\omega_0 T_{0\mathbf{k}} e^{-ikx} + \omega_0 T^{*}_{0\mathbf{k}} e^{ikx}),\end{aligned}$$

which satisfies the Klein-Gordon equation. This field is an infinite array of quantized oscillators. Its Hamiltonian density equation is,

$$H = 1/2[(\partial_0 \varphi)^2 + (\nabla \varphi)^2 + \omega_0^2 \varphi^2], \tag{80}$$

It is a quantized field.

Field Quantization

The transition to a quantum field can be done via canonical quantization. Physical observables shall be promoted to operators.

$$N_{\mathbf{k}} = \omega_0^2 \, T_{0\mathbf{k}}^{\dagger} \, T_{0\mathbf{k}}, \tag{81}$$

$$a_{\mathbf{k}} = \omega_0 \, T_{0\mathbf{k}}, \tag{82}$$

$$a_{\mathbf{k}}^{\dagger} = \omega_0 \, T_{0\mathbf{k}}^{\dagger}, \tag{83}$$

such that $N_{\mathbf{k}} = a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$. The Hamiltomian density becomes

$$H = \frac{1}{V} \sum_{\mathbf{k}} \omega_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}, \tag{84}$$

which have structures resemble a zero-spin bosonic field.

- Extension to non-relativistic limit and Schrodinger equation is straight forward.

Non-Relativistic Limit

In the non-relativistic limit, we will define:

$$\psi_{\mathbf{k}} = \frac{\omega_0 T_{0\mathbf{k}}}{\sqrt{V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_c t + \chi)} \approx \left[\frac{\omega_0^2}{\sqrt{V}} e^{i(\omega_0 t + \chi)}\right] \zeta_{\mathbf{k}}, \tag{85}$$

where $e^{i\chi}$ is an arbitrary phase factor. $\psi_{\mathbf{k}}$ is a solution for the Schrödinger equation of a free particle, $-i\partial\psi_{\mathbf{k}}/\partial t = (2m)^{-1}\nabla^2\psi_{\mathbf{k}}$. The superposition principle holds such that

$$\psi = e^{i\chi} \sum_{\mathbf{k}} \frac{\omega_0 T_{0\mathbf{k}}}{\sqrt{V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_c t)}, \tag{86}$$

is also a solution.

Probability Density

$$\psi_{\mathbf{k}}^{*}\psi_{\mathbf{k}} = \frac{\omega_0^2 T_{0\mathbf{k}}^{*} T_{0\mathbf{k}}}{V} = \frac{n_{\mathbf{k}}}{V}, \tag{87}$$

is a particle number density. The amplitude $\alpha_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}/\sqrt{V}$ in Eq. (85) is a probability amplitude.

$$\psi = e^{i\chi} \sum_{\mathbf{k}} \frac{\omega_0 T_{0\mathbf{k}}}{\sqrt{V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_c t)}, \tag{88}$$

The introduction of the arbitrary phase factor $e^{i\chi}$ does not change the the probability density $\psi^{*}\psi$ or the result that ψ satisfies the Schrödinger equation. The theory developed with wave functions ψ shall be invariant under global phase transformation χ but the relative phase factors are physical. The overall phase of ψ is unobservable.

Reasons against Time as Operator

- According to Pauli's reasoning, a time operator t satisfies $[H, t] = i$. Let $H\Psi_E = E\Psi_E$, and then we have $He^{i\alpha t}\Psi_E = (E + \alpha)e^{i\alpha t}\Psi_E$, where α is an arbitrary constant. However, $(E + \alpha)$ is an eigenvalue of $e^{i\alpha t}\Psi_E$ which implies time cannot be an operator because contradict the fact that the Hamiltonian spectrum must be positive.

Temporal Vibration Operator

Base on the condition $\omega_0^2 T_0^* T_0 = n$,

$$T_{\mathbf{k}} = \frac{\omega}{\omega_0} T_{0\mathbf{k}} = \frac{\omega}{\omega_0^2} a_{\mathbf{k}}, \tag{89}$$

$$T_{\mathbf{k}}^\dagger = \frac{\omega}{\omega_0} T_{0\mathbf{k}}^\dagger = \frac{\omega}{\omega_0^2} a_{\mathbf{k}}^\dagger, \tag{90}$$

satisfying commutation relations,

$$[T_{\mathbf{k}}, T_{\mathbf{k}'}^\dagger] = \frac{\omega^2}{\omega_0^4} \delta_{\mathbf{k}\mathbf{k}'}, \tag{91}$$

$$[T_{\mathbf{k}}, T_{\mathbf{k}'}] = [T_{\mathbf{k}}^\dagger, T_{\mathbf{k}'}^\dagger] = 0. \tag{92}$$

In the Heisenberg picture, $T_{\mathbf{k}}(t)$ and $T_{\mathbf{k}}^\dagger(t)$ evolve over time as:

$$\frac{d}{dt} T_{\mathbf{k}}(t) = i[H_{\mathbf{k}}(t), T_{\mathbf{k}}(t)] = -i\omega T_{\mathbf{k}}(t) \quad \rightarrow \quad T_{\mathbf{k}}(t) = T_{\mathbf{k}}(0) e^{-i\omega t}, \tag{93}$$

$$\frac{d}{dt} T_{\mathbf{k}}^\dagger(t) = i[H_{\mathbf{k}}(t), T_{\mathbf{k}}^\dagger(t)] = i\omega T_{\mathbf{k}}^\dagger(t) \quad \rightarrow \quad T_{\mathbf{k}}^\dagger(t) = T_{\mathbf{k}}^\dagger(0) e^{i\omega t}. \tag{94}$$

Temporal Vibration Operator

ζ is a real scalar field that can be applied to obtain the temporall vibrations of matter in a bosonic field. It can be rewritten in terms of $T_{\mathbf{k}}$ and $T_{\mathbf{k}}^{\dagger}$ as:

$$\zeta(\mathbf{x}) = \sum_{\mathbf{k}} \sqrt{\frac{\omega_0}{2\omega^3}} [T_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} + T_{\mathbf{k}}^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}}]. \tag{95}$$

The temporal vibrations field operator is the time derivative of $\zeta(\mathbf{x})$ by applying Eq. (61),

$$t_d(\mathbf{x}) = \zeta_t(\mathbf{x}) = \partial_0 \zeta(\mathbf{x}) = \sum_{\mathbf{k}} -i \sqrt{\frac{\omega_0}{2\omega}} [T_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} - T_{\mathbf{k}}^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}}], \tag{96}$$

Temporal Vibration Operator

The Lagrangian density for the real scalar field is:

$$\mathcal{L} = \frac{\bar{\rho}_m \omega_0^2}{2}[(\partial_0 \zeta)^2 - (\nabla \zeta)^2 - \omega_0^2 \zeta^2], \tag{97}$$

where

$$\bar{\rho}_m = \frac{\omega_0}{V}, \tag{98}$$

is a mass density constant of the system. Hence, the conjugate momenta of $\zeta(\mathbf{x})$ is:

$$\eta(\mathbf{x}) = \frac{\partial \mathcal{L}}{\partial[\partial_0 \zeta(\mathbf{x})]} = -i\bar{\rho}_m \omega_0^2 \sum_{\mathbf{k}} \sqrt{\frac{\omega_0}{2\omega}}[T_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} - T_{\mathbf{k}}^{\dagger} e^{i\mathbf{k}\cdot\mathbf{x}}] = \bar{\rho}_m \omega_0^2 \zeta_t(\mathbf{x}). \tag{99}$$

Temporal Vibration Operator

Satisfy the equal-time commutation relations:

$$[\zeta(t,\mathbf{x}),\eta(t,\mathbf{x}')] = i\delta(\mathbf{x}-\mathbf{x}'), \tag{100}$$

$$[\zeta(t,\mathbf{x}),\zeta(t,\mathbf{x}')] = [\eta(t,\mathbf{x}),\eta(t,\mathbf{x}')] = 0, \tag{101}$$

Similarly,

$$[\zeta(t,\mathbf{x}),\zeta_t(t,\mathbf{x}')] = (\bar{\rho}_m\omega_0^2)^{-1}\delta(\mathbf{x}-\mathbf{x}'), \tag{102}$$

$$[\zeta_t(t,\mathbf{x}),\zeta_t(t,\mathbf{x}')] = 0. \tag{103}$$

The conjugate momenta of $\zeta(\mathbf{x})$, describe the temporal vibrations in a bosonic field .

Temporal Vibration Operator

- A temporal oscillator has displacement in either the positive or negative temporal direction relative to the external time t.
- Intrinsic time t_f is the summation of the temporal vibration ζ_t and the external time t
- The intrinsic time of a particle travels at an average rate of the external time
- The difference between the external time and intrinsic time is measured by the temporal vibration operator
- $\zeta(\mathbf{x})$, $\zeta_t(\mathbf{x})$ and $\eta(\mathbf{x})$ are self adjoint operators
- The temporal vibration and the Hamiltonian do not form a conjugate pair, no commutation relation with the semi-bounded Hamiltonian that restrict the spectrum of the temporal vibration operator to be bounded
- External time t remains a parameter called for by Pauli's theorem

1/2 Spin with Temporal Vibration

Consider a spin-1/2 particle wave in the non-relativistic limit,

$$\zeta_{1/2}(t) = \frac{e^{-i\beta(t)}}{\omega_0^2}, \tag{104}$$

$$\beta(t) = \omega_0 t = -\hat{\phi}(t)/2. \tag{105}$$

β is the phase of the temporal oscillation; $\hat{\phi}$ is the intrinsic angle of rotation about the z axis that 'runs through' the particle observed.

- The first equality, $\beta(t) = \omega_0 t$, states that the wave has vibrations in proper time
- The plane wave is normalized and the particle observed is at rest
- The second equality, $\omega_0 t = -\hat{\phi}(t)/2$, implies the particle is rotating about its own z-axis
- Its angular velocity, $\partial\hat{\phi}(t)/\partial t = -2\omega_0$, is twice as fast as the temporal oscillation

1/2 Spin with Temporal Vibration

Eq. (104) can be rewritten,

$$\zeta_{1/2}(t) = \frac{e^{i\hat{\phi}(t)/2}}{\omega_0^2}. \tag{106}$$

- Requires the rotation of $\hat{\phi} = 4\pi$ to return $\zeta_{1/2}$ to its original state
- Temporal oscillation and particle rotation can be visualized as two wheels
- A larger 'time wheel' and a smaller 'particle wheel'
- Turning one of the wheels will drive the other to turn
- Require 'particle wheel' to rotate two turns before 'time wheel' complete one
- Temporal oscillation and particle rotation are connected by $\beta = -\hat{\phi}/2$

1/2 Spin with Temporal Vibration

$\zeta_{1/2}$ can also be expressed as a function of $\hat{\phi}$,

$$\zeta_{1/2}(\hat{\phi}) = \frac{e^{i\hat{\phi}/2}}{\omega_0^2}. \tag{107}$$

- An angular momentum is observed when there is a periodic structure to its wave function
- Defining the intrinsic angular momentum operator as,

$$S_z = -i\frac{\partial}{\partial\hat{\phi}}, \tag{108}$$

  and taking $\zeta_{1/2}(\hat{\phi})$ as its eigenfunction, the corresponding eigenvalue is 1/2 which is the intrinsic angular momentum of the particle
- The intrinsic angular momentum of this particle is a 'pure' quantum phenomenon

1/2 Spin with Temporal Vibration

Reverse the direction of the particle's rotation, $\hat{\phi} \to -\hat{\phi}$, i.e.

$$\zeta_{-1/2}(\hat{\phi}) = \frac{e^{-i\hat{\phi}/2}}{\omega_0^2}. \tag{109}$$

- Taking $\zeta_{-1/2}(\hat{\phi})$ as an eigenfunction of S_z, the corresponding eigenvalue is $-1/2$
- The particle can have two different spin states with intrinsic angular momentum of $\pm 1/2$ along its z axis
- Two-component complex-valued spinors, i.e.

$$|\zeta_{1/2}(t)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{e^{-i\omega_0 t}}{\omega_0^2}, \tag{110}$$

$$|\zeta_{-1/2}(t)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{e^{-i\omega_0 t}}{\omega_0^2}, \tag{111}$$

Experimenting with Neutrino

A neutrino with $m = 2\mathrm{eV}$ ($\omega_0 = 3.04 \times 10^{15}\mathrm{s}^{-1}$ and $\tilde{T}_0 = 1/\omega_0 = 3.29 \times 10^{-16}\mathrm{s}$),

$$E = 1\mathrm{Mev} \quad \mathring{T}_n = 2.3\times10^{-13}\mathrm{s}, \quad |\mathring{\mathbf{X}}_n| = 7.0\times10^{-3}\mathrm{cm}, \quad \omega_p = 6.1\times10^{9}\mathrm{s}^{-1}, \tag{112}$$

$$E = 1\mathrm{Gev} \quad \mathring{T}_n = 7.4 \times 10^{-12}\mathrm{s}, \quad |\mathring{\mathbf{X}}_n| = 0.22\mathrm{cm}, \quad \omega_p = 6.1 \times 10^{6}\mathrm{s}^{-1}. \tag{113}$$

- Temporal oscillation can affect the rate of change for the intrinsic properties of a particle, e.g. decay rate of an unstable particle, neutrino oscillation, etc.
- Intrinsic time in average equals external time. However, during a cycle of oscillation, it can deviate from the average. The standard deviation is not zero.
- **Neutrino can be an interesting candidate because of its extreme light weight**

Conclusion

- Schwarzschild field
- Born's postulate
- Why matter field need to be quantized
- Pauli's theorem
- Einstein's mass-energy relation
- $1/2$ spin particle

The End