Proper Time Oscillator

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May 23, 2020

Abstract

By restoring the symmetry between time and space, we reconciled the properties of a zero-spin quantum field from a matter field that has vibrations in time. This quantized real scalar field obeys the Klein-Gordon equation and Schrödinger equation. Also, the particles observed are oscillators in proper time. After neglecting all quantum effects, we show that a proper time oscillator can mimic a point mass at rest in general relativity. The spacetime outside this proper time oscillator is static and satisfies the Schwarzschild solution. Time can have a more dynamic role than what we have previously considered.

1 Introduction

Nature, in general, prefers symmetries. For example, Maxwell saw the need to add an extra term to bring symmetry to his electrodynamic equations. This ingenious idea ultimately led to the full unification of electrodynamics and optics. Another well known example is the symmetry between space and time. Nowadays, we understand that spacetime shall be weaved as unity. However, despite the preference for symmetries, the treatment of time and space in quantum theory is asymmetrical, which has created a constellation of problems when we try to unify the fundamental theories.

In quantum theory, time is postulated as a parameter. It is not treated as an operator based on the reasoning put forward by Pauli. On the other hand, when we study a system like a classical or quantum simple harmonic oscillator, oscillations are considered only in the spatial directions. Thus, if space and time are to be treated on the same footing as stipulated by the theory of relativity, can there be oscillation in the temporal direction? In fact, if we consider $E = mc^2$, it looks like an energy equation for an oscillator where c can be interpreted as the rate of time. Therefore, is it possible that the energy of mass has something to do with oscillation in time? Although it is theoretically feasible to construct an oscillator that has a vibration in time by taking analogy to the classical oscillator, can it be related to a matter field? More importantly, how does this time oscillator affects the surrounding spacetime geometry?

In this essay, we will try to answer the above questions based on the results obtained in refs. [1–7]. Our analyses show that a matter field with vibrations in time has the same properties of a zero-spin quantum field. The particle in this real scalar field is a proper time oscillator, and its energy is on shell. Also, the spacetime outside this time oscillator is static and satisfies the Schwarzschild solution if we neglect all the quantum effects. Interestingly, time can have a more dynamic role than what we have previously considered.

2 Matter Field with Vibrations in Time

This section outlines the basic results obtained in refs. [2, 4–7]. To begin, let us consider the background coordinates $(t, \mathbf{x})$ in a flat spacetime. Time in this background is the 'external time' as measured by clocks stationary at spatial infinity that are not coupled to the system under investigation. Taking time as a dynamical variable, we can construct a Lorentz covariant plane wave that has vibrations of matter in both the temporal and spatial directions, i.e.

$$t_f = t + T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = t + \mathrm{Re}(\zeta_t), \tag{1}$$

$$\mathbf{x}_f = \mathbf{x} + \mathbf{X}\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = \mathbf{x} + \mathrm{Re}(\zeta_{\mathbf{x}}), \tag{2}$$

where

$$\zeta_t = -iTe^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad \zeta_{\mathbf{x}} = -i\mathbf{X}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{3}$$

Analogous to the spatial vibration amplitude $\mathbf{X}$, the amplitude for vibration in time, T, is defined as the maximum difference between the time of matter inside the wave, t_f, and the external time, t. Therefore, if matter inside the plane wave carries a clock measuring its internal time, an inertial observer outside will see the matter's clock vibrates with time, t_f, as related to his own clock measuring time, t. In other words, we have assumed the matter's internal clock is running at a varying rate relative to the inertial observer's clock at spatial infinity. The amplitude $(T, \vec{X})$ is a 4-vector such that $T^2 = T_0^2 + |\vec{X}|^2$, where T_0 is an amplitude with vibration in proper time.

We can further summarize these vibrations with a single function,

$$\zeta = \frac{T_0}{\omega_0}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{4}$$

The vibrations ζ_t and $\zeta_{\mathbf{x}}$ can be written as $\zeta_t = \partial_0\zeta$ and $\zeta_{\mathbf{x}} = -\nabla\zeta$ respectively. As we shall note, ζ satisfies the wave equation, $\partial_u\partial^u\zeta + \omega_0^2\zeta = 0$. This equation is similar to the Klein-Gordon equation, but ζ is not necessarily a quantized field. So, why is the energy of this system ought to be quantized?

For a plane wave that can have multiple numbers of particles with mass m in a cube with volume V, the Hamiltonian density is,

$$\mathcal{H} = \frac{m\omega_0^2}{2V}[(\partial_0\zeta^*)(\partial_0\zeta) + (\nabla\zeta^*)\cdot(\nabla\zeta) + \omega_0^2\zeta^*\zeta], \tag{5}$$

In particular, the Hamiltonian density of a plane wave with only temporal vibrations is $\mathcal{H}_0 = m\omega_0^2T_0^*T_0/V$. Since there is no vibration in the spatial directions ($\omega = \omega_0, |\mathbf{k}| = 0$ and $|\mathbf{X}| = 0$), the Hamiltonian density $\mathcal{H}_0$ shall correspond to the certain internal energy of matter at rest. However, we have only considered the matter with mass m in this simple harmonic oscillating system with no other energy or force field. Therefore, we can only consider $\mathcal{H}_0$ as an internal mass-energy density arises from the proper time vibrations of matter.

For a wave with only one particle, the energy inside volume V is $E = m\omega_0^2 T_0^* T_0$ for the Hamiltonian density $\mathcal{H}_0$. If the energy of this harmonic oscillator is the internal mass-energy of matter, it can only be observed as the energy of mass m, which is on shell, i.e.,

$$E = m = m\omega_0^2 T_0^* T_0. \tag{6}$$

Therefore, a point mass m can only oscillate with a particular amplitude, $\mathring{T}_0 = 1/\omega_0$. As mentioned in the Introduction, the internal mass-energy $E = mc^2$ does look like an equation of an oscillator.

The internal time $\mathring{t}_f$ with amplitude $\mathring{T}_0 = 1/\omega_0$ of the point mass's internal clock is:

$$\mathring{t}_f = t - \frac{\sin(\omega_0 t)}{\omega_0}. \tag{7}$$

Such a particle will travel along a near time-like geodesic if the measuring clock is not sensitive enough to detect the oscillation. On the other hand, if the oscillation of a particle is slow enough, we can observe its properties under different time rates within a cycle. For example, an unstable particle will have different decay rates observed at different phases of the oscillation.

The internal time rate of this oscillator relative to the external time is $\partial\tilde{t}_f/\partial t = 1 - \cos(\omega_0 t)$. Not only the average of this time rate is 1, but its value is also bounded between 0 and 2, which is positive. Therefore, the internal time of the oscillator moves only forward. It cannot go back to its past.

As we shall recall, a particle's energy is supposed to be on shell, and a fraction of its energy cannot be observed. If this is the case, how can we explain the uniform Hamiltonian density obtained? In fact, we cannot unless we consider the system as a probability wave. A probability density $\bar{\rho}$ can be assigned for the observation of a particle at a particular location, i.e. $\bar{\mathcal{H}}_0 = \bar{\rho} m$. The Hamiltonian density can be obtained by averaging the results from many measurements. Based on this probability density, we can relate the plane wave ζ and the quantum wave function ψ, i.e., $\psi = \zeta\sqrt{\omega_0^3\omega/V}e^{i\omega_0 t}$. The basic properties of a quantum wave can then be developed by following the common procedures in standard quantum mechanics.

As it is well known in quantum theory, when the Klein-Gordon equation is treated as a single particle equation in a relativistic theory, one will encounter the difficulties of negative energy solutions. Since ζ satisfies an equation similar to the Klein-Gordon equation, we expect the system with vibrations in space and time shall have the same properties of a zero spin matter field in quantum theory. Following the same procedures in quantum field theory, the transition of a classical field to a quantum field can be done via canonical quantization. In other words, we can treat $\zeta(\vec{x})$ and $\mathcal{H}$ as operators. We can also relate ζ with the bosonic field φ, i.e., $\varphi(\vec{x}) = \zeta(\vec{x})\sqrt{\omega_0^3/V}$. The result is that the real scalar field with vibrations of matter in time can have the same physical properties as a zero-spin bosonic field.

There are two other properties that we will only state the results in here. First, ψ is not required to have the same phase as ζ. Although the overall phase of a wave function is unobservable, it does not hinder the interpretations that a matter wave can have vibrations in time. Second, the internal time t_f can be treated as a self-adjoint operator without contradicting Pauli's theorem. Interested readers can find our analyses in refs. [2, 5].

3 Gravitational Field of a Time Oscillator

This section outlines the basic results obtained in refs. [1, 3]. To study the gravitational field around the proper time oscillator, we will neglect all quantum effects and treat the oscillator as a classical system that can stay at the origin of a coordinate system, $r = 0$. We will also consider the proper time oscillation as a part of the spacetime geometry. As discussed earlier, the local time at the coordinate of the oscillator is driven by the mass-energy to propagate with varying rates. This time oscillation is geometrically different from the assumed flat spacetime at spatial infinity. Therefore, if the spacetime manifold is smooth and continuous, its structures cannot be flat outside the proper time oscillation.

Being a wave itself, the oscillation in proper time from Eq. (7) can be decomposed into Fourier series of plane waves. This can be done by the superposition of Lorentz covariant plane waves which we will define as $\xi_{t\mathbf{k}} = -iT_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. However, we need to point out that this plane wave is not the same $\zeta_{t\mathbf{k}}$ as defined in the previous section for describing the vibrations of matter in time. Here, we will use $\xi_{t\mathbf{k}}$ to describe the time vibration as a part of the spacetime geometry. We shall also bear in mind that the Lorentz covariant plane wave has a component with vibrations in space, i.e., $\xi_{x\mathbf{k}}$. Therefore, apart from the proper time oscillation at $r = 0$, there can be other vibrations in the spatial directions. These spatial oscillations will only be revealed after we carried out the Fourier decomposition of the time oscillation. The results of our analysis are summarized as follow:

At $r = 0$,

$$\mathring{t}_f(t, 0) = t - \frac{\sin(\omega_0 t)}{\omega_0}, \tag{8}$$

$$\mathring{r}_f(t, 0) = 0. \tag{9}$$

At $r = \epsilon/2 \to 0$,

$$\mathring{t}_f(t, \epsilon/2) = t, \tag{10}$$

$$\mathring{r}_f(t, \epsilon/2) = \epsilon/2 + \Re_\infty \cos(\omega_0 t) \text{ with } \Re_\infty \to \infty. \tag{11}$$

As shown, the point mass at rest has two oscillating components: the proper time oscillator at $r = 0$ and the radial oscillations about a thin shell with $r = \epsilon/2 \to 0$. They are simple harmonic oscillators. Based on our knowledge about simple harmonic oscillating systems, their total energies are conserved over time. The rest mass system as a whole, therefore, shall have a symmetry under time translation as demanded by the Noether's theorem. Outside these oscillators, there are no other vibrations, and the spacetime is a vacuum.

Matter cannot have a simple harmonic spatial motion that has an amplitude with infinite magnitude. This motion will violate the principles of relativity by allowing the superluminal transfer of energy. The radial oscillation, therefore, cannot be interpreted as a vibration that can carry an observer through space. Instead, we shall study the effects of this radial oscillation on an observer that is stationary at $r = \epsilon/2$.

As shown in Eqs. (10) and (11), it is the clock of a fictitious observer $\underline{O}$ oscillating about $r = \epsilon/2$ that synchronize with the clock of an inertial observer O at spatial infinity. In this fictitious frame, $\underline{O}$ is an inertial observer. An observer O_+ on the shell with radius $r = \epsilon/2$ will have an oscillation $\underline{r}_f$ relative to the fictitious inertial observer $\underline{O}$, i.e.

$$\underline{r}_f(t, \epsilon/2) = -\mathring{r}_f(t, \epsilon/2) + \epsilon/2 = -\Re_\infty \cos(\omega_0 t). \tag{12}$$

Therefore, O_+ is under the constant effects of a fictitious oscillation while remaining at rest relative to O at spatial infinity. These effects shall impact the temporal/spatial measurements and the metric on the thin shell.

Instead of working directly with the radial oscillations about $r = \epsilon/2$, we can consider an infinitesimally thin spherical shell Σ with radius $\breve{r}(> 2m)$. Relative to this shell, an observer $\breve{O}$ stationary at $r = \breve{r}$ has a fictitious displacement $\underline{r}_f$ relative to a fictitious observer $\underline{O}$,

$$\underline{r}_f(t, \breve{r}) = -\mathring{r}_f(t, \breve{r}) + \breve{r} = -\breve{\Re} \cos(\omega_0 t), \tag{13}$$

Although $\breve{O}$ is stationary relative to O at spatial infinity, it is under the effects as if $\breve{O}$ is oscillating in the fictitious frame of $\underline{O}$. The properties of these radial oscillations with amplitude $\breve{\Re}$ are analogous to those about $r = \epsilon/2$, except the magnitude of the oscillation amplitude $\breve{\Re}$ is finite. These radial oscillations shall have effects on the metric of the surrounding spacetime geometry.

Our analyses show that the metric on the surface of this thin shell is $ds^2 = [1 - \breve{I}]dt^2 - [1 - \breve{I}]^{-1}dr^2 - \breve{r}^2 d\Omega^2$, where $\breve{I} = \breve{\Re}^2\omega_0^2$. The vacuum space–time υ^+ outside this spherical thin shell Σ (a time-like hypersurface) is the Schwarzschild spacetime, i.e.

$$ds^2 = [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]dt^2 - [1 - \frac{\breve{r}\breve{\Re}^2\omega_0^2}{r}]^{-1}dr^2 - r^2 d\Omega^2. \tag{14}$$

As shown, this metric is the same for a massive thin shell with mass m if we set,

$$m = \frac{\breve{r}\breve{\Re}^2\omega_0^2}{2} \quad \text{or} \quad \breve{\Re} = \sqrt{\frac{2}{\breve{r}m}}. \tag{15}$$

In Eq. (15), the amplitude $\breve{\Re}$ remains well defined if the shell is contracted until it reaches a radius $\breve{r} = \epsilon/2 \to 0$. According to Birkhoff's theorem, the thin shell can be contracted without affecting the outside spacetime as long as the equivalence mass m is kept as a constant. Also, the contracted thin shell with radius $\breve{r} = \epsilon/2$ has an amplitude $\breve{\Re} \to \infty$. It is the same shell engendered by the proper time oscillator with amplitude $\Re_\infty$ and angular frequency ω_0 as discussed earlier. Therefore, the spacetime outside the proper time oscillator is static and satisfies the Schwarzschild solution. A proper time oscillator can mimic a point mass at rest in general relativity if we neglect all the quantum effects.

4 Conclusions and Discussions

"Matter tells spacetime how to curve, and curved spacetime tells matter how to move." General relativity is a theory that can describe the effects of matter on spacetime. However, it lacks an explanation of why matter can curve spacetime. As we have tried to demonstrate in this essay, the answer to this question may involve quantum theory. For instance, particles are treated as sets of coupled oscillators with their own de Broglie's internal clock in quantum theory.

Interestingly, the proper time oscillators meet these criteria. If a particle has something to do with the proper time oscillator as we have discussed in Section 2, this could explain how the intrinsic structures of matter can influence the spacetime geometry.

The model can establish a more direct correlation between spacetime and matter. However, we shall bear in mind that the quantum effects have been neglected when we developed the gravitational field of a proper time oscillator. To fully understand the gravitational properties of a proper time oscillator, it will require the development of a complete quantum gravity theory.

Finally, there is one more question that we shall explore. How can we measure the effects of this time oscillation in an experiment? To answer this question, we need to mention a property of the proper time oscillator that we have not discussed so far. We find that a proper time oscillator will have oscillations in the temporal and spatial directions when it is in motion [2, 5]. At a higher speed, the oscillator will have a lower frequency and larger amplitudes of oscillation. Consequently, it will be easier to detect the effects of the oscillations at a higher speed. For example, the temporal oscillation can theoretically affect the rate of how the neutrinos' flavor and mass eigenstates are mixed over time. This effect will be amplified at higher speed and energy. Another possibility is to detect the spatial oscillations associated with the temporal oscillation of a particle. Two neutrinos with the same initial velocity can reach a target at slightly different times, depending on the relative phase of their oscillations. In theory, this deviation can be observable by repeated measurements of the neutrinos' arrival times at a detector. Because of their extremely light weight, neutrinos can be projected to a very high speed, which can amplify the oscillations for measurements.

To demonstrate the magnitude of the oscillations, we will quote our analysis in ref. [2] for a neutrino with $m = 2eV$:

$$E = 1\text{Gev} \quad \Rightarrow \quad \mathring{T} = 7.4 \times 10^{-12}\text{s}, \quad |\mathring{\mathbf{X}}| = 0.22\text{cm}, \quad \omega_p = 6.1 \times 10^{6}\text{s}^{-1}. \tag{16}$$

These quantities are not outrageously small/large for a neutrino moving at near light speed. Neutrinos can be interesting candidates for investigating the effects of the proper time oscillator because of their extreme light weight.

References

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