Some non-trivial aspects of Poincaré and CPT invariance of flavor vacuum

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Contents

Motivations

- Particles states in relativistic quantum field theory (QFT) are usually assumed to belong to unitary irreducible representations of Poincaré group[1]
- Flavor states, not having a definite mass, do not fit in such a scheme
- In QFT construction, flavor states can be viewed as excitations of a flavor vacuum, where Poincaré and CPT symmetry are spontaneously broken. This does not affect flavor oscillations[2]

[1] V. Bargmann, E.P. Wigner, Proc. Natl. Acad. Sci. U.S.A. **34**, 211 (1948).
[2] M. Blasone, P. Jizba, N.E Mavromatos and L.S., arXiv:2002.11072 [hep-th].

Poincaré group and flavor states

Poincaré transformations

A generic Poincaré transformation can be written as:

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + b^{\mu}$$

An infinitesimal spacetime translation is:

$$T(\delta b) = \mathbb{1} + i\delta b^{\mu} P_{\mu}$$

P_{μ} is the four-momentum operator. An infinitesimal proper-Lorentz transformation is:

$$\Lambda(\delta\omega) = \mathbb{1} - \frac{i}{2}\delta\omega_{\mu\,\nu} J^{\mu\,\nu}$$

$J^{\mu\,\nu}$ is the angular momentum tensor.

Casimir of Poincaré group

Poincaré group has two Casimir invariants:

$$M^2 \equiv P_\sigma P^\sigma \qquad W^2 = W_\sigma W^\sigma$$

where

$$W_\sigma = -\frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\,J^{\mu\nu}\,P^\rho$$

is the Pauli–Lubansky operator.

Particle states are usually assumed to belong to the unitary irreducible representations of the Poincaré group. For massive states:

$$M^2\,|k^2,s,\sigma\rangle = m_\sigma^2\,|k^2,s,\sigma\rangle\,, \quad W^2\,|k^2,s,\sigma\rangle = -m_\sigma^2\,s(s+1)\,|k^2,s,\sigma\rangle$$

Here σ indicates other internal quantum numbers.

Flavor fields and flavor states

Let us now consider the Lagrange density[3]

$$\mathcal{L}(x) \;=\; \partial_\mu \boldsymbol{\varphi}_f^\dagger(x) \partial^\mu \boldsymbol{\varphi}_f(x) \;-\; \boldsymbol{\varphi}_f^\dagger(x)\, M^2\, \boldsymbol{\varphi}_f(x)$$

where

$$\boldsymbol{\varphi}_f(x) \;=\; \begin{bmatrix} \varphi_A(x) \\ \varphi_B(x) \end{bmatrix}, \qquad M^2 \;=\; \begin{bmatrix} m_A^2 & m_{A\,B}^2 \\ m_{A\,B}^2 & m_B^2 \end{bmatrix}$$

it is clear that we cannot take flavor states as elements of irreducible representations of the Poincaré group. Otherwise:

$$M^2\,|k_\sigma, \sigma\rangle \;=\; m_\sigma^2\,|k_\sigma, \sigma\rangle\,, \qquad \sigma \;=\; A, B$$

[3]M. Blasone, A. Capolupo, O. Romei and G. Vitiello, Phys. Rev. D **63**, 125015 (2001).

Flavor states and Poincaré group

- A possibility is to consider flavor states as belonging to irreducible representations of an extended Poincaré group. Coleman–Mandula theorem[4] tells us that the only reasonable extension is $T^{3,1} \rtimes O(3,1) \times G$, where G is an internal flavor symmetry group.
- A construction of this type has been recently proposed[5]
- Another possibility is that Poincaré symmetry were spontaneously broken when field mixing is dynamically generated [6]. This last possibility originates from string theory (D-foam models[7] [8])

[4]S. Coleman and T. Mandula, Phys. Rev. **159**, 159 (1967).
[5]A.E Lobanov, Ann. Phys. **403**, 82 (2019).
[6]M. Blasone, P. Jizba, N.E Mavromatos and L.S., in preparation.
[7]N.E. Mavromatos, S. Sarkar and W. Tarantino, Phys. Rev. D **80**, 084046 (2009).
[8]M.Blasone, P. Jizba, N.E. Mavromatos and L.S., Phys. Rev. D. **100**, 045027 (2019).

Scalar field mixing in QFT

Mass fields

The above Lagrangian can be diagonalized:

$$\begin{bmatrix}\varphi_A(x)\\ \varphi_B(x)\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\varphi_1(x)\\ \varphi_2(x)\end{bmatrix}$$

where $\tan 2\theta = 2\, m_{AB}^2 / \left(m_B^2 - m_A^2\right)$. Mass fields are expanded as usual:

$$\varphi_j(x) = \int \frac{\mathrm{d}^3\mathbf{k}}{2\omega_{\mathbf{k},j}(2\pi)^3}\left[a_{\mathbf{k},j}\, e^{-i\omega_{\mathbf{k},j}t} + b^\dagger_{-\mathbf{k},j}\, e^{i\omega_{\mathbf{k},j}t}\right] e^{i\mathbf{k}\cdot\mathbf{x}}$$

where the annihilation and creation operators satisfy:

$$\left[a_{\mathbf{k},i}\,,\, a^\dagger_{\mathbf{p},j}\right] = \left[b_{\mathbf{k},i}\,,\, b^\dagger_{\mathbf{p},j}\right] = 2\omega_{\mathbf{k},i}\,(2\pi)^3\,\delta(\mathbf{k}-\mathbf{p})\,\delta_{ij}\,.$$

Flavor fields quantization (1)

One can expand flavor fields as:

$$\varphi_\sigma(x) \;=\; \int \frac{\mathrm{d}^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3}\, \left[a_{\mathbf{k},\sigma}(t)\, e^{-i\omega_{\mathbf{k},\sigma} t} \;+\; b^\dagger_{-\mathbf{k},\sigma}(t)\, e^{i\omega_{\mathbf{k},\sigma} t}\right] e^{i\mathbf{k}\cdot\mathbf{x}}$$

with $\omega_{\mathbf{k},\sigma} = \sqrt{|\mathbf{k}|^2 + \mu_\sigma^2}$ and μ_σ are mass parameters which have to be specified. The flavor ladder operators are:

$$\begin{bmatrix} a_{\mathbf{k},A} \\ b^\dagger_{-\mathbf{k},A} \\ a_{\mathbf{k},B} \\ b^\dagger_{-\mathbf{k},B} \end{bmatrix} = \begin{bmatrix} c_\theta\, \rho^{\mathbf{k}}_{A1} & c_\theta\, \lambda^{\mathbf{k}}_{A1} & s_\theta\, \rho^{\mathbf{k}}_{A2} & s_\theta\, \lambda^{\mathbf{k}}_{A2} \\ c_\theta\, \lambda^{\mathbf{k}*}_{A1} & c_\theta\, \rho^{\mathbf{k}*}_{A1} & s_\theta\, \lambda^{\mathbf{k}*}_{A2} & s_\theta\, \rho^{\mathbf{k}*}_{A2} \\ -s_\theta\, \rho^{\mathbf{k}}_{B1} & -s_\theta\, \lambda^{\mathbf{k}}_{B1} & c_\theta\, \rho^{\mathbf{k}}_{B2} & c_\theta\, \lambda^{\mathbf{k}}_{B2} \\ -s_\theta\, \lambda^{\mathbf{k}*}_{B1} & -s_\theta\, \rho^{\mathbf{k}*}_{B1} & c_\theta\, \lambda^{\mathbf{k}*}_{B2} & c_\theta\, \rho^{\mathbf{k}*}_{B2} \end{bmatrix} \begin{bmatrix} a_{\mathbf{k},1} \\ b^\dagger_{-\mathbf{k},1} \\ a_{\mathbf{k},2} \\ b^\dagger_{-\mathbf{k},2} \end{bmatrix}$$

where $c_\theta \equiv \cos\theta,\ s_\theta \equiv \sin\theta,\ \rho^{\mathbf{k}}_{\sigma j} \;=\; |\rho^{\mathbf{k}}_{\sigma j}| e^{i(\omega_{\mathbf{k},\sigma} - \omega_{\mathbf{k},j})t}$, $\lambda^{\mathbf{k}}_{\sigma j} \;=\; |\lambda^{\mathbf{k}}_{\sigma j}| e^{i(\omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},j})t}$

Flavor fields quantization (2)

and

$$|\rho^{\mathbf{k}}_{\sigma j}| \;=\; \frac{1}{2}\left(\frac{\omega_{\mathbf{k},\sigma}}{\omega_{\mathbf{k},j}}+1\right), \qquad |\lambda^{\mathbf{k}}_{\sigma j}| \;=\; \frac{1}{2}\left(\frac{\omega_{\mathbf{k},\sigma}}{\omega_{\mathbf{k},j}}-1\right)$$

This is a canonical transformation:

$$\left[a_{\mathbf{k},\sigma}(t)\,,\,a^{\dagger}_{\mathbf{p},\rho}(t)\right] \;=\; \left[b_{\mathbf{k},\sigma}(t)\,,\,b^{\dagger}_{\mathbf{p},\rho}(t)\right] \;=\; 2\omega_{\mathbf{k},\sigma}\,(2\pi)^3\,\delta(\mathbf{k}-\mathbf{p})\,\delta_{\sigma\rho}$$

The flavor vacuum is annihilated by flavor annihilation operators at a fixed time t:

$$a_{\mathbf{k},\sigma}(t)\,|0(t)\rangle_{A,B} \;=\; b_{\mathbf{k},\sigma}(t)\,|0(t)\rangle_{A,B} \;=\; 0$$

Flavor states

Flavor states:

$$|a_{\mathbf{k},\sigma}(t)\rangle \;\equiv\; a^{\dagger}_{\mathbf{k},\sigma}(t)|0(t)\rangle_{A,B}$$

These are eigenstates of the flavor charges

$$Q_\sigma(t) \;=\; i\int d^3\mathbf{x}\,\varphi^\dagger_\sigma(x)\;\overleftrightarrow{\partial_0}\;\varphi_\sigma(x)$$

at fixed time t

$$Q_\sigma(t)\,|a_{\mathbf{k},\sigma}(t)\rangle \;=\; |a_{\mathbf{k},\sigma}(t)\rangle$$

Poincaré and CPT symmetry breaking

Space translations

The generator of space translations, so that $T(\mathbf{b})\varphi_\sigma(t,\mathbf{x})T^{-1}(\mathbf{b}) = \varphi_\sigma(t,\mathbf{x}+\mathbf{b})$, is:

$$T(\mathbf{b}) \;\equiv\; \exp\left(i\,\mathbf{b}\cdot\mathbf{P}\right)$$

where the momentum operator is

$$P_i \;=\; \sum_{\sigma=A,B} \int \frac{\mathrm{d}^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3}\, k_i\, \left(a^\dagger_{\mathbf{k},\sigma}(t)\, a_{\mathbf{k},\sigma}(t) + b^\dagger_{\mathbf{k},\sigma}(t)\, b_{\mathbf{k},\sigma}(t)\right)$$

These annihilate the flavor vacuum

$$P_i|0(t)\rangle_{A,B} \;=\; 0$$

Hamiltonian in the flavor representation

The Hamiltonian reads

$$H \;=\; \sum_{\sigma,\tau} \int \frac{d^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3} \Big[w^{\mathbf{k}}_{\sigma\tau} \Big(a^\dagger_{\mathbf{k},\sigma}(t)\, a_{\mathbf{k},\tau}(t) + b^\dagger_{\mathbf{k},\sigma}(t)\, b_{\mathbf{k},\tau}(t) \Big)$$

$$+ \; y^{\mathbf{k}}_{\sigma\tau} \Big(a^\dagger_{\mathbf{k},\sigma}(t)\, b^\dagger_{-\mathbf{k},\tau}(t) + b_{-\mathbf{k},\sigma}(t)\, a_{\mathbf{k},\tau}(t) \Big) \Big]$$

where $w^{\mathbf{k}}_{\sigma\tau}, y^{\mathbf{k}}_{\sigma\tau}$ are non-zero coefficients. Then

$$H|0(t)\rangle_{A,B} \;=\; \sum_{\sigma,\tau} \int \frac{d^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3}\, y^{\mathbf{k}}_{\sigma\tau}\, |a_{\mathbf{k},\sigma}(t)\rangle \otimes |b_{-\mathbf{k},\tau}(t)\rangle$$

Flavor vacuum manifold

Flavor vacua at different times are given by:

$$|0(t)\rangle_{A,B} \; = \; T(t)\,|0\rangle_{A,B}$$

with

$$T(t) \;\equiv\; \exp\left(i\,H\,t\right)$$

They form a manifold of states with equal total charge

$$Q \; = \; Q_A(t) + Q_B(t)$$

By Fabri–Picasso theorem[9] flavor Fock spaces at different times are unitarily inequivalent.

[9] E. Fabri, L. E. Picasso, Phys. Rev. Lett. **16**, 408 (1966).

Oscillation formula

In fact

$$ {}_{A,B}\langle 0|H^2|0\rangle_{A,B} \;=\; V\,{}_{A,B}\langle 0|H\,\mathcal{H}(0)|0\rangle_{A,B} $$

which diverges for $V \to \infty$. Flavor oscillation formula:

$$ \mathcal{Q}_{\sigma\to\rho}(t,t_0) \;=\; {}_{A,B}\langle a_{\mathbf{k},\sigma}(t_0)|Q_\rho(t)|a_{\mathbf{k},\sigma}(t_0)\rangle_{A,B} $$

This depends only on time differences:

$$ \mathcal{Q}_{\sigma\to\rho}(t,t_0) \;=\; \mathcal{Q}_{\sigma\to\rho}(t-t_0) $$

Rotations

The angular momentum operator is:

$$\mathbf{L} \;=\; \int \frac{\mathrm{d}^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3} \left[a^\dagger_{\mathbf{k},\sigma}(t)\; (\mathbf{k}\times\nabla_{\mathbf{k}})\; a_{\mathbf{k},\sigma} \;+\; b^\dagger_{\mathbf{k},\sigma}(t)\; (\mathbf{k}\times\nabla_{\mathbf{k}})\; b_{\mathbf{k},\sigma}(t)\right]$$

Generator of rotations:

$$R(\boldsymbol{\vartheta}) \;=\; \exp\left(-i\,\boldsymbol{\vartheta}\cdot\mathbf{L}\right)$$

This is a symmetry of vacuum:

$$\mathbf{L}\,|0(t)\rangle_{A,B} \;=\; 0$$

Boosts

Generator of boosts along the l-th axis:

$$K_l = \Big[x^0 \int d^3\mathbf{x} \left(\boldsymbol{\pi}_f^\dagger(x)\, \partial^l \boldsymbol{\varphi}_f(x) + \partial^l \boldsymbol{\varphi}_f^\dagger(x)\, \boldsymbol{\pi}_f(x) \right)$$

$$- \int d^3\mathbf{x}\, x^l \left(\boldsymbol{\pi}_f^\dagger(x)\, \boldsymbol{\pi}_f(x) + \nabla \boldsymbol{\varphi}_f^\dagger(x) \cdot \nabla \boldsymbol{\varphi}_f(x) + \boldsymbol{\varphi}_f^\dagger(x)\, M^2\, \boldsymbol{\varphi}_f(x) \right) \Big]$$

A generic boost can be written in terms of the unitary generator:

$$U(L) = \exp\left(-i \sum_{l=1}^{3} \xi^l\, K_l \right)$$

so that

$$U(L)\, \varphi_\sigma(x)\, U^{-1}(L) = \varphi_\sigma(x')$$

This is not a flavor vacuum symmetry: Vacuum symmetry is $E(3)$.

Flavor wave-packets

Ladder operators transform as:

$$U(L)\, a_{\mathbf{k},\sigma}(t)\, U^{-1}(L) \;=\; a_{L\mathbf{k},\sigma}\,(t')$$

$$U(L)\, b_{\mathbf{k},\sigma}(t)\, U^{-1}(L) \;=\; b_{L\mathbf{k},\sigma}\,(t')$$

Flavor wave-packets:

$$|a_\sigma(y)\rangle \;\equiv\; \int \frac{\mathrm{d}^3\mathbf{k}}{2\omega_{\mathbf{k},\sigma}(2\pi)^3}\, e^{-iky}\, f(\mathbf{k})\, a_{\mathbf{k},\sigma}(y_0)|0(y_0)\rangle_{A,B}$$

If $f(\mathbf{k})$ is Lorentz invariant:

$$|a'_\sigma(y)\rangle \;\equiv\; U(L)|a_\sigma(y)\rangle \;=\; |a_\sigma\,(y')\rangle$$

Invariance of oscillation formula

Covariat form of flavor oscillation formula:

$$\mathcal{J}^{\mu}_{\sigma\to\rho}(x-y) \;=\; \langle a_\sigma(y)|J^{\mu}_{\rho}(x)|a_\sigma(y)\rangle$$

where $J^{\mu}_{\rho}(x)$ are the flavor currents:

$$J^{\mu}_{\rho}(x) \;\equiv\; i\varphi^{\dagger}_{\rho}(x)\,\overset{\leftrightarrow}{\partial^{\mu}}\,\varphi_{\rho}(x)$$

In the primed Lorentz frame:

$$\langle a'_\sigma(y)|J^{\mu}_{\rho}(x')|a'_\sigma(y)\rangle \;=\; \langle a_\sigma(y')|J^{\mu}_{\rho}(x')|a_\sigma(y')\rangle \;=\; \mathcal{J}^{\mu}_{\sigma\to\rho}(x'-y')$$

Oscillation formula is Lorentz invariant!

Parity

Parity transformation:

$$P\,\varphi_\sigma(x)\,P^{-1} \;=\; \eta_{\sigma,P}\,\varphi_\sigma(\tilde{x})$$

where $\tilde{x}=(t,-\mathbf{x})$ and $\eta_{\sigma,P}$ is a phase. Ladder operators transform as:

$$P\,a_{\mathbf{k},\sigma}(t)\,P^{-1} \;=\; \eta_{\sigma,P}\,a_{-\mathbf{k},\sigma}(t) \qquad P\,b_{\mathbf{k},\sigma}(t)\,P^{-1} \;=\; \eta^*_{\sigma,P}\,b_{-\mathbf{k},\sigma}(t)$$

$$P\,a^\dagger_{\mathbf{k},\sigma}(t)\,P^{-1} \;=\; \eta^*_{\sigma,P}\,a^\dagger_{-\mathbf{k},\sigma}(t) \qquad P\,b^\dagger_{\mathbf{k},\sigma}(t)\,P^{-1} \;=\; \eta_{\sigma,P}\,b^\dagger_{-\mathbf{k},\sigma}(t)$$

The flavor vacuum is invariant under parity transformation:

$$P\,|0(t)\rangle_{A,B} \;=\; |0(t)\rangle_{A,B}$$

Charge conjugation

The charge conjugation transformation:

$$C\,\varphi_\sigma(x)\,C^{-1} \;=\; \eta_{\sigma,C}\,\varphi^\dagger_\sigma(x)$$

Transformations of creation and annihilation operators follow:

$$\begin{aligned} C\,a_{\mathbf{k},\sigma}(t)\,C^{-1} &= \eta_{\sigma,C}\,b_{\mathbf{k},\sigma}(t) \qquad & C\,b_{\mathbf{k},\sigma}(t)\,C^{-1} &= \eta^*_{\sigma,C}\,a_{\mathbf{k},\sigma}(t) \\ C\,a^\dagger_{\mathbf{k},\sigma}(t)\,C^{-1} &= \eta^*_{\sigma,C}\,b^\dagger_{\mathbf{k},\sigma}(t) \qquad & C\,b^\dagger_{\mathbf{k},\sigma}(t)\,C^{-1} &= \eta_{\sigma,C}\,a^\dagger_{\mathbf{k},\sigma}(t) \end{aligned}$$

The flavor vacuum is invariant under charge conjugation:

$$C\,|0(t)\rangle_{A,B} \;=\; |0(t)\rangle_{A,B}$$

Time reversal

Time reversal transformation:

$$T\,\varphi_\sigma(x)\,T^{-1} \;=\; \eta_{\sigma,_T}\,\varphi_\sigma(-\tilde{x})\,, \tag{1}$$

Transformations of creation and annihilation operators follow:

$$\begin{aligned} T\,a_{\mathbf{k},\sigma}(t)\,T^{-1} &= \eta_{\sigma,_T}\,a_{-\mathbf{k},\sigma}(-t) \qquad & T\,b_{\mathbf{k},\sigma}(t)\,T^{-1} &= \eta^*_{\sigma,_T}\,b_{-\mathbf{k},\sigma}(-t) \\ T\,a^\dagger_{\mathbf{k},\sigma}(t)\,T^{-1} &= \eta^*_{\sigma,_T}\,a^\dagger_{-\mathbf{k},\sigma}(-t) \qquad & T\,b^\dagger_{\mathbf{k},\sigma}(t)\,T^{-1} &= \eta_{\sigma,_T}\,b^\dagger_{-\mathbf{k},\sigma}(-t) \end{aligned}$$

This is not a flavor vacuum symmetry. However, flavor oscillation formula is invariant:

$$\mathcal{Q}_{\sigma\to\rho}(-t) \;=\; \mathcal{Q}_{\sigma\to\rho}(t)$$

CPT

CPT transformation:

$$\Theta \equiv C\,P\,T$$

Transformation of creation and annihilation operators:

$$\begin{aligned} \Theta\, a_{\mathbf{k},\sigma}(t)\,\Theta^{-1} &= \eta_\sigma\, b_{\mathbf{k},\sigma}(-t) & \qquad \Theta\, b_{\mathbf{k},\sigma}(t)\,\Theta^{-1} &= \eta^*_\sigma\, a_{\mathbf{k},\sigma}(-t) \\ \Theta\, a^\dagger_{\mathbf{k},\sigma}(t)\,\Theta^{-1} &= \eta^*_\sigma\, b^\dagger_{\mathbf{k},\sigma}(-t) & \qquad \Theta\, b^\dagger_{\mathbf{k},\sigma}(t)\,\Theta^{-1} &= \eta_\sigma\, a^\dagger_{\mathbf{k},\sigma}(-t) \end{aligned}$$

This is not a flavor vacuum symmetry. However, flavor oscillations are CPT invariants:

$$\mathcal{J}^\mu_{\sigma\to\rho}(x) \;=\; \mathcal{J}^\mu_{\bar{\sigma}\to\bar{\rho}}(-x) \qquad (2)$$

Conclusions and perspectives

Conclusions

- Flavor states do not belong to unitarily irreducible representations of the Poincaré group
- In QFT flavor states can be constructed as excitations of flavor vacuum
- While Lagrangian is Poincaré and then CPT invariant, the flavor vacuum is not: SSB of Poincaré down to $E(3)$
- SSB of CPT symmetry
- Flavor oscillation formula is Poincaré and CPT invariant

Perspectives

- Analysis of the fermion case
- Extension to three flavors: Explicit CP symmetry breaking
- Connection with Standard Model Extension[10]
- Explicit mechanism of Poincaré SSB in D-foam models[11].

[10] V.A. Kostelecky and M. Mewes, Phys. Rev. D **69**, 016005 (2004).
[11] N.E Mavromatos, S. Sarkar and W. Tarantino, Mod. Phys. Lett. A **28**, 1350045 (2013).

Thank you for the attention!