The kinematic form of relativistic quantum mechanics and its possible underlying structure

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The 12th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields

1 - 4 June 2020 ♦ Czech Technical University in Prague

Formulas of quantum mechanics and Planck's constant

In Quantum mechanics formulations, Planck's constant is responsible for the dynamical forms.

So without Planck's constant, The energy converted to frequency, the momentum converted to wavelength (wavenumber),…
Then, the Quantum mechanics formula without Planck's constant may show a kinematic structure.

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi$$

Schrödinger equation for free particle

$$i\frac{\partial\psi}{\partial t} = -\frac{c^2}{2\omega}\nabla^2\psi$$

Schrödinger equation for free particle without $\hbar$

But there must be a physical meaning for that structure in space and time.

Quantum Theory without Planck's Constant

Within quantum mechanics, there have been a few attempts to derive the quantum mechanics equations without Planck's constant, for different reasons. (Barut, 1992 and Ralston, 2012).

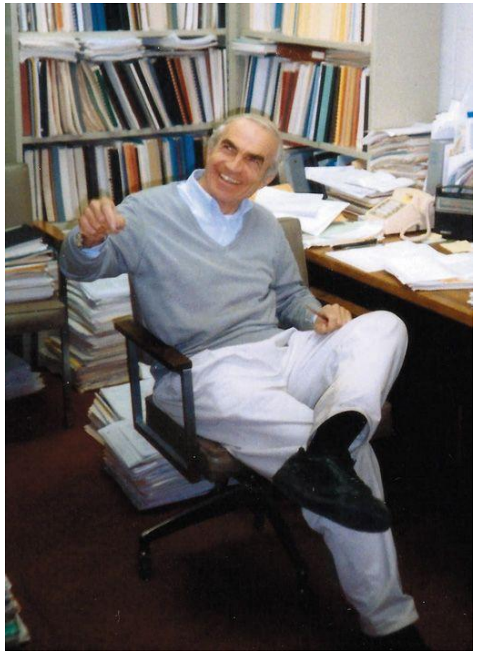
For Ralston, the Planck's constant "*is unobservable except as a constant of human convention. Despite long reference to experiment, review shows that Planck's constant cannot be obtained from the data of Ryberg, Davisson and Germer, Compton, or that used by Planck himself. In the new approach Planck's constant is tied to macroscopic conventions of Newtonian origin, which are dispensable*". (Ralston, 2012).

Barut, A. (1992). Formulation of wave mechanics without the Planck constant h, Physics Letters A. **171**, Issues 1–2, 30 November 1992, pp. 1–2.

Ralston, J. (2012). Quantum Theory without Planck's Constant, arXiv:1203.5557v1 [hep-ph] 25 Mar 2012.

Barut's approach

Barut tried to formulate quantum mechanics without the parameters Planck's constant , mass and charge as a "pure wave theory" in terms of the frequencies alone. *"This is more directly related to experiments where one measures frequency differences rather than energies. Different quantum systems are then characterized by an intrinsic proper frequency "*(Barut, 1992).



Asım Orhan Barut
(1926 – 1994)

Barut, A. (1992). Formulation of wave mechanics without the Planck constant h, Physics Letters A. **171**, Issues 1–2, 30 November 1992, pp. 1–2.

Barut treatment of mass

- Deal with the mass via theory of mass where it is shown that localized solution of the massless wave equations with an internal frequency (Ω) move like relativistic particles with the dispersion relation:

$$\left(\frac{\omega}{c}\right)^2 = k^2 + \left(\frac{\Omega}{c}\right)^2$$

The frequency Ω gives the system the inertia mass.

Barut's wave

The wave equation

$$\Box \phi = 0$$

The non-spreading solutions of the wave equation are of the forms:

- In the rest frame

$$\phi = F(r) \exp -i\Omega t$$

Complex form ?

- In a moving frame

$$\phi = F\left[(\gamma\,(x - vt)^2 + y^2 + z^2)^{1/2}\right] exp - i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$$

$$\omega = \gamma\,\Omega\;\boldsymbol{k} = \gamma\,\Omega\,\frac{\boldsymbol{\beta}}{c}$$

Barut's forms

The Schrödinger equation for the Coulomb problem

$$i\frac{\partial\psi}{\partial(ct)} = -\frac{1}{2(\Omega/c)}\Delta\psi + \sqrt{\frac{2\omega_0}{\Omega}}\frac{1}{r}\Psi$$

The Dirac-Coulomb equation is

$$i\frac{\partial\psi}{\partial(ct)} = -i\vec{\alpha}\cdot\nabla\psi + \beta\left(\frac{\Omega}{c}\right)\psi + \sqrt{\frac{2\omega_0}{\Omega}}\frac{1}{r}\Psi$$

Coulomb

The Dirac equation for free particle

$$i\frac{\partial\psi}{\partial(ct)} = \left[-i\,\alpha\cdot\nabla + \beta\left(\frac{\Omega}{c}\right)\right]\psi.$$

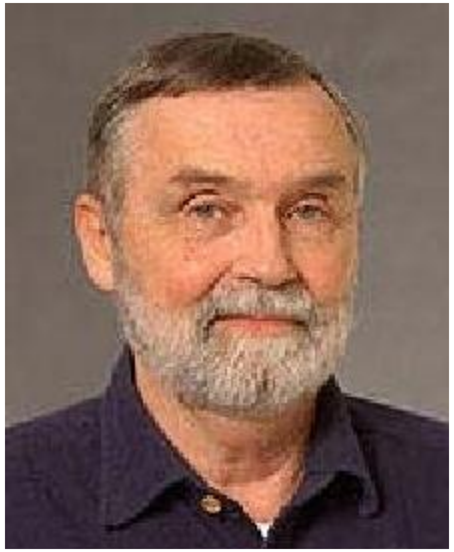
The proportionality constant $\hbar$ is taken from experiment.

The kinematic model

The concept of the kinematic model that is related to the complex phase factor has been considered by Hestenes during the nineties of the last century.

Within his geometric algebra, Hestenes proposed many concepts, like :

- Hestenes, D., *Space-Time Algebra*, Gordon & Breach, New York (1966).
- Hestenes, D., The Zitterbewegung Interpretation of Quantum Mechanics, *Found. Phys.*, **20**, 1213–1232 (1990a).
- Hestenes, D., On Decoupling Probability from Kinematics in Quantum Mechanics, P. F. Fougere (ed.), Maximum Entropy and Bayesian Methods, Kluwer Academic Publishers, 161-183 (1990b).



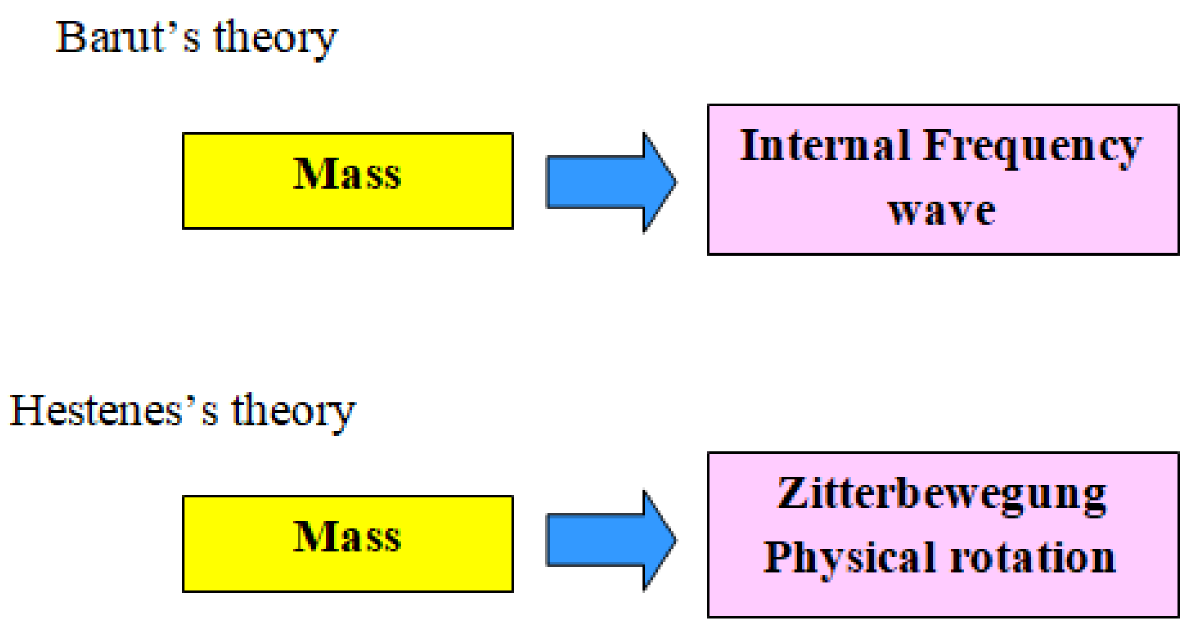
David Hestenes (1933-)

Kinematic motion models are mathematical models that describe the motion of objects without consideration of forces.

- The imaginary *i* can be interpreted as a representation of the electron spin.
- Dirac theory describes a kinematics of electron motion. The kinematical rotation is not necessary to be related to the pair of positive and negative energy states.
- The complex phase factor literally represents a *physical* rotation, the *zitterbewegung* rotation.
- The complex phase factor is the main feature which the Dirac wave function shares with its nonrelativistic limit. Schrödinger wave function inherits the relativistic nature.

The serious concept in Hestenes proposal is the kinematic origin of the complex phase factor and the physical rotation (Zitterbewegung), but, there is no experimental evidence to support these ideas yet.

Mass and the kinematic models



Is there a relationship between the kinematic system and *i* and $\hbar$?

❖ According to Hestenes:

- The imaginary *i is related to the spin (kinematic system) !*
- The complex phase factor literally represents a *physical* rotation, the *zitterbewegung* rotation *(kinematic system)*
- Dirac theory describes a kinematics of electron motion.
- The complex phase factor is the main feature in the relativistic and nonrelativistic wave function forms.

❖ According to Barut the Planck's constant ($\hbar$) is the proportionality constant for a *kinematic system*

Relativistic quantum mechanics forms

The Klein-Gordon equation is a second-order time-differential equation,

$$i^2\hbar^2\frac{\partial^2\psi}{\partial t^2} = [c^2P^2 + m^2c^4]\psi$$

derivative of order 2

whereas Dirac's equation is a first-order time-differential equation,

$$i\hbar\frac{\partial\psi}{\partial t} = (c\boldsymbol{\alpha}\cdot\boldsymbol{P} + \beta mc^2)\psi$$

derivative of order 1

For a free particle, the trial solution has the form:

$$\psi_D(x,t) = u_D(x,t)\exp\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{x} - Et)$$

derivative of order 0

Formulation without Planck's constant

- Klein-Gordon and Dirac equations contain the constant ***ħ*** on both sides of the equations.
- These equations contain the imaginary ***i*** but on one side of the equations.
- The complex phase factor of the wave function is multiplied and divided by ***ħ*** and multiplied by the imaginary ***i***.

One can cancel Planck's constant from each side without any mathematical problem. Then, rewrite these equations without ***ħ*** .

Formulation without Planck's constant

In considering the de Broglie, Planck, and Einstein equations:

$$\hat{p} = -i\hbar\nabla$$

$$E = \omega\,\hbar$$

$$\omega = m\,c^2/\hbar$$

The angular frequency of these equations (ω) is related to the frequency of energy of mass.

Relativistic quantum mechanics forms without $\hbar$

The Klein-Gordon becomes:

$$i^2 \frac{\partial^2 \psi}{\partial t^2} = [-c^2(\boldsymbol{\nabla} \cdot \boldsymbol{\nabla}) + \omega^2]\psi$$

derivative of order 2

Dirac equation becomes:

$$i \frac{\partial \psi}{\partial t} = (-ic\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta\omega)\psi$$

derivative of order 1

Dirac wave function becomes:

$$\psi_D(x,t) = u_D(x,t) \exp i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$$

derivative of order 0

Even though Planck's constant is not presented, these equations are still equations of quantum mechanics.

The system of particle-wave?

The parameters of the complex phase are in kinematical form and may form a system as shown in the table.

	Wave vector	Angular frequency
Particle		$\frac{E}{\hbar} = \omega_p$
wave	$\frac{\boldsymbol{p}}{\hbar} = \frac{1}{\boldsymbol{\lambda}} = \boldsymbol{k}_w$	

$$r_w = \frac{\lambda}{2\pi} = \frac{1}{k_w}$$

$$Complex\ face\ factor \equiv \exp i\,(k \cdot x - \omega t),$$

	Radius	Angular frequency
Particle	$r_p = 0$	ω_p
wave function	r_w	$\omega_w = 0$

Looking for kinematical system

- Dirac form (without $\hbar$) is similar to Barut's form.
- These formulas are not related to a certain kinematical model as in the Barut's attempt.

What type of kinematical system are these equations related to?

	Wave vector	Angular frequency
Particle		$\frac{E}{\hbar} = \omega_p$
wave	$\frac{\boldsymbol{p}}{\hbar} = \frac{1}{\boldsymbol{\lambda}} = \boldsymbol{k}_w$	

Circles theory

- Since 2007, a kinematical model is being developed to study the complexity (appearing of imaginary *i* in physics equations) as a cause of physical causes. (Sanduk, 2007, 2012, 2008a, 2008b, 2019,…)

- This work has no relationship with quantum mechanics. But it was triggered by Three wave hypothesis (TWH).

- It is a kinematical theory based on a study of the motion of rolling circles. It may be termed as "circles theory". The timeline of the circles theory has been shown in Fig. 2.

Covariant æther & quantum mechanics

Kostro in 1978 proposed the Three Wave Model (TWM), and is based on two frameworks:

- Einstein's theory of relativity, where the existence of covariant æther is assumed.
- The Paris school interpretation of quantum mechanics (de Broglie's works), and

In the TWM, three waves are associated with a micro-object:

1-internal nondispersive standing wave,

2-superluminal wave (the de Broglie wave), and

3-subluminal wave.

Ludwik Kostro

Kostro, L., A wave model of the elementary particle, a three waves hypothesis, unpublished paper sent to L. de Broglie in the French version (1978).

Kostro, L., A three-wave model of the elementary particle, Physics letters, **107A**, number 9,7, pp. 429-434 (1985).

Three Wave Hypothesis

During the 1980s, the TWM has been developed and presented as the Three Wave Hypothesis (TWH) by Horodecki. The TWH implies that a particle having mass is an intrinsically spatially and temporally extended nonlinear wave phenomenon , and considers three waves:

1-the Compton,

2-de Broglie, and

3-dual waves.

It is clear that it has a fluid base.

Ryszard Horodecki

Horodecki, R., De Broglie wave and its dual wave, Phys. Lett. 87A, pp. 95-97 (1981).
Horodecki, R., Superluminal singular dual wave, Lett. Novo Cimento 38, 509-511 (1983).

The problem of the bevel gear

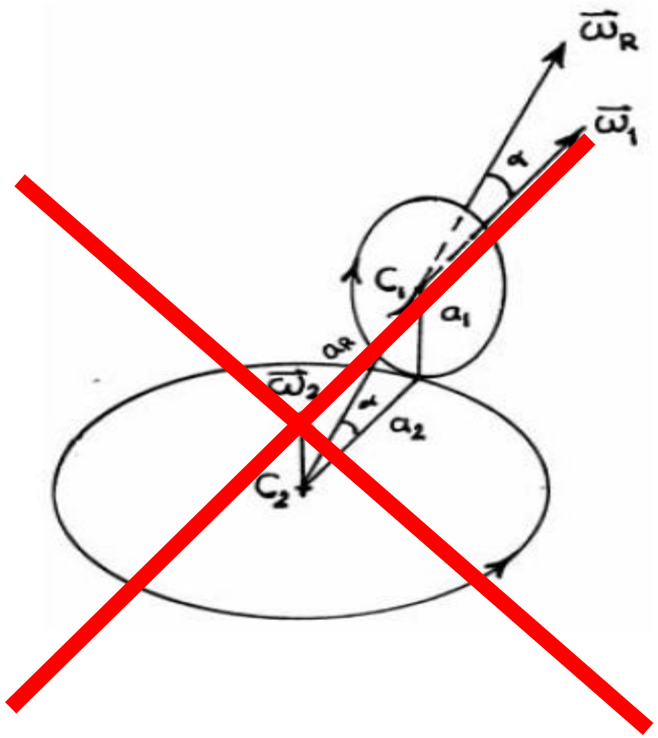
Here one may ask,

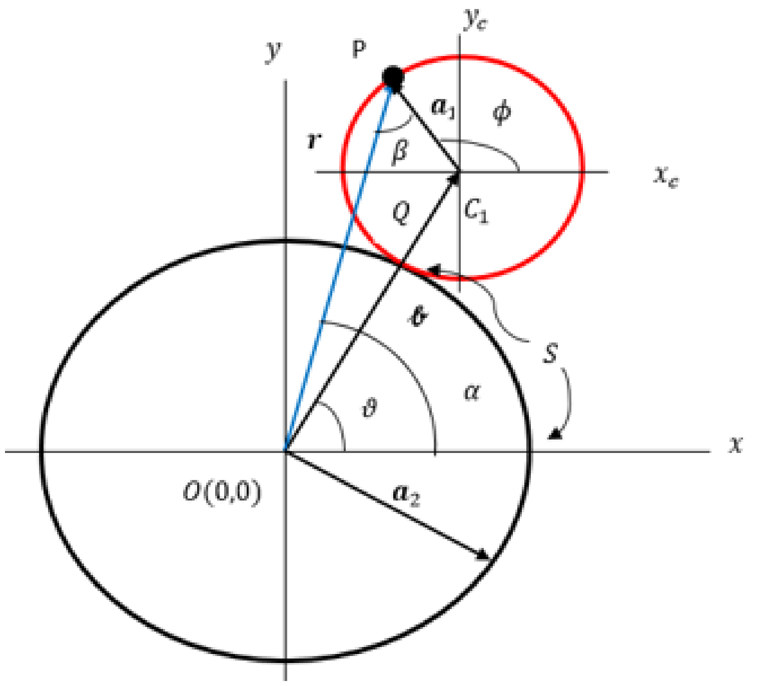
- *does this real kinematical model stand behind the concept of complex form in quantum mechanics*?; or
- *is there any relationship between the system of the two rolling circles with the complex function*?

Between 2009-2012, I tried to get answers, but I did not convince in what I did. There is no problem with the concept of the two rolling circles. The problem is with the perpendicular structure (covariant form)!!

$$\omega_C = \pm\left(\omega_B{}^2 + \omega_{C\circ}{}^2\right)^{\frac{1}{2}}$$

In 2012, I started with two rolling circles in a real plane. This system is not related to the covariant ether.





Based on TWH of the covariant æther and the de Broglie's works . 2007

Mathematical model has no relation with covariant æther. 2012

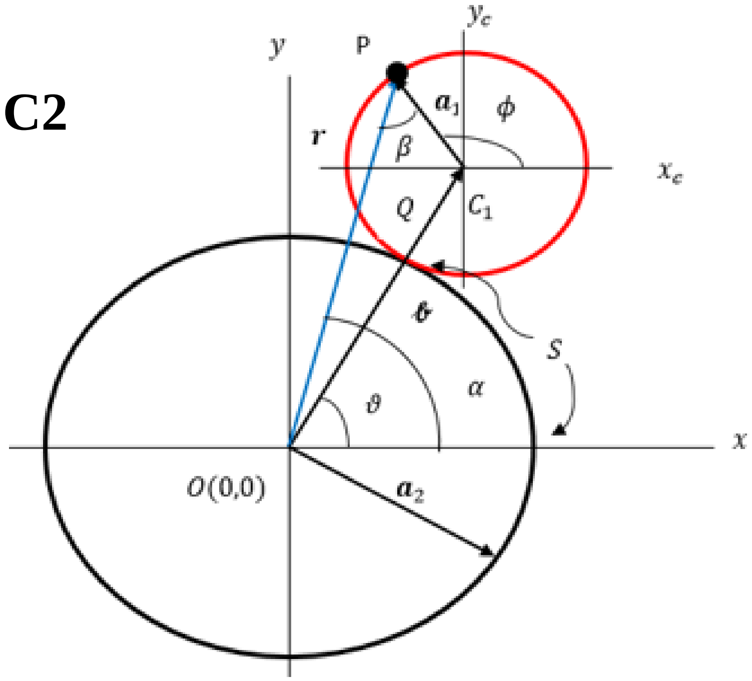
Sanduk, M. I., Three Wave Hypothesis, Gear Model and the Rest Mass, arXiv:0904.0790 [physics.gen-ph] (2009).

1-Real rolling circles system

The position vector ($\boldsymbol{r}$) of a point P in a system of two rolling circles is (Sanduk, 2012, 2018):

In relative to C2

$$\boldsymbol{r} = \boldsymbol{b}\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-\sin^2(\vartheta - \phi + \beta) + \left(\frac{a_1}{\boldsymbol{b}}\right)^2}\right\}$$



For generality, let $a_1 < a_2$. The ratio of the system is:

$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1$$

Sanduk, M., A kinematic model for a partially resolved dynamical system in a Euclidean plane, Journal of Mathematical Modelling and Application, Vol. 1, No.6, 40-51 (2012).

Sanduk, M., An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether, *International Journal of Quantum Foundations* 4 (2018) 173 - 198

Table of the mathematical system in the external world

	Radius	Angular frequency
Small circle	a_{1m}	ω_{1m}
Large circle	a_{2m}	ω_{2m}

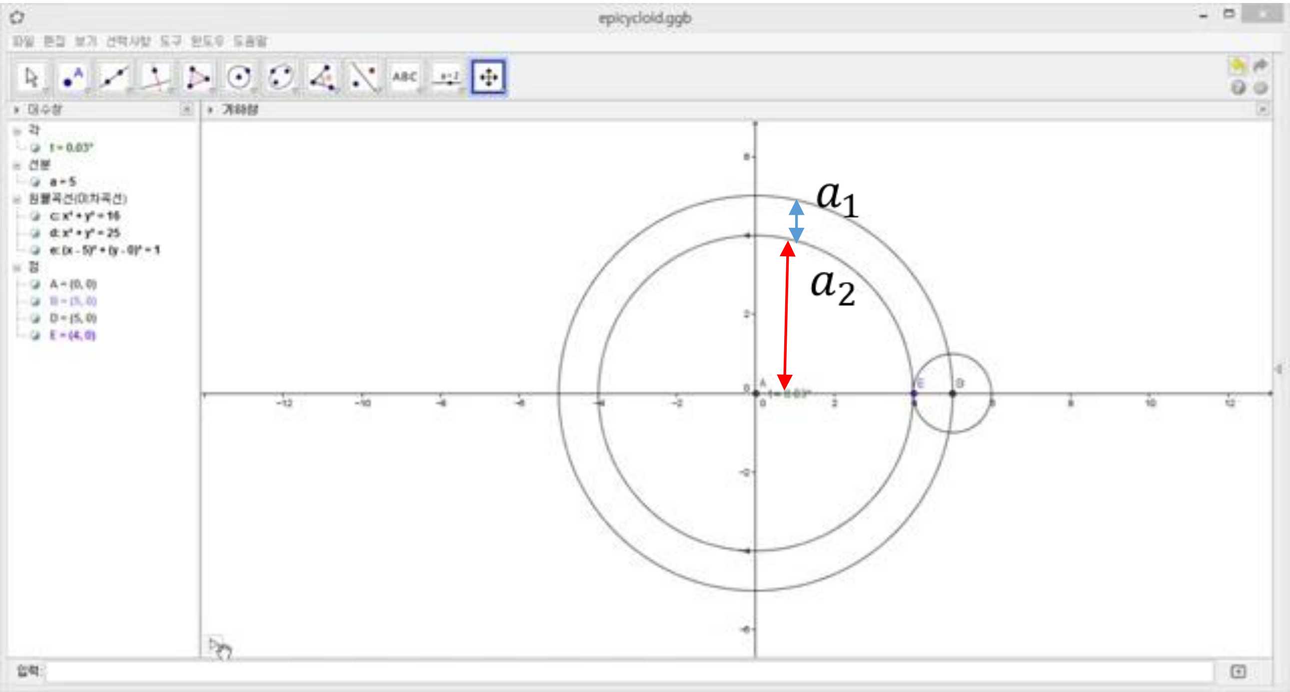
$$\boldsymbol{r} = \boldsymbol{b}\left\{\cos\left(\vartheta - \phi + \beta\right) \pm \sqrt{-sin^2(\vartheta - \phi + \beta) + \left(\frac{a_1}{\boldsymbol{b}}\right)^2}\right\}$$

$$X = \frac{a_1}{b}$$

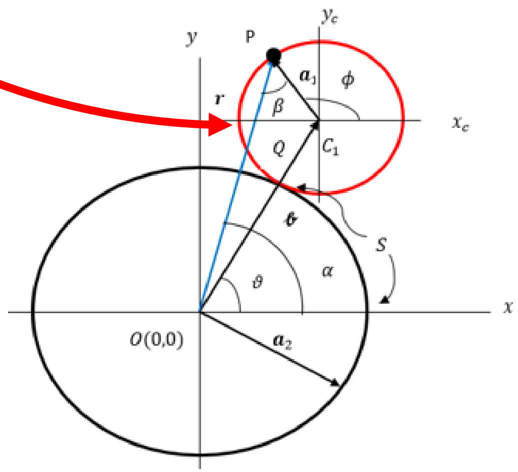
$$\boldsymbol{r} = \left(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X}\right)\left\{\cos\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\}$$

Two rolling circles system
Epicycloid

$$r = b\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-sin^2(\vartheta - \phi + \beta) + \left(\frac{a_1}{b}\right)^2}\right\}$$



$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1$$



$$\frac{a_2}{a_1} = \frac{\omega_1}{\omega_2} = \mu = 4$$

Velocity Equation

Differentiation of $\boldsymbol{r}$ with time shows a velocity equation of point P in real plane

$$\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t}$$

$$= \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\Big\{cos\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big)$$

$$\pm \sqrt{-sin^2\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) + X}\Big\} + (\boldsymbol{a}_2$$

$$+ \boldsymbol{b}\sqrt{X})\Big\{-(-\omega_1 + \omega_\beta) sin\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big)$$

$$\pm \left(\frac{1}{2}\right)\frac{(-2)(-\omega_1 + \omega_\beta)\, sin(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t) cos\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \cdots}{\sqrt{-sin^2\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) + X}}$$

In relative to C2

Acceleration equation

$$\Theta = \boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t$$

In relative to C2

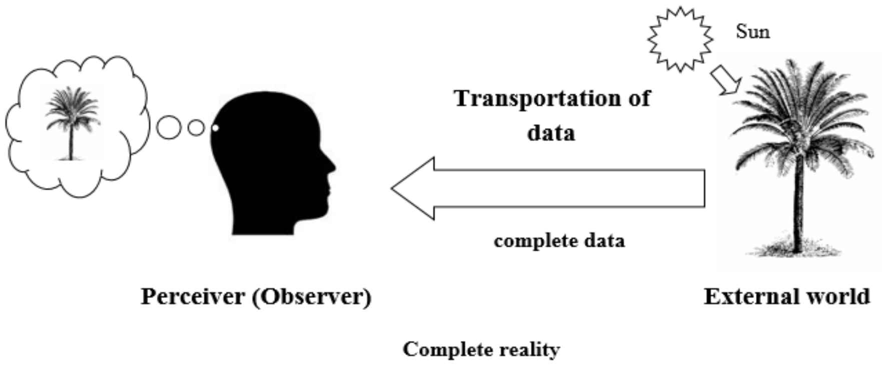
$$\frac{\partial^2 \boldsymbol{r}(r,t,X)}{\partial t^2}$$
$$= \frac{\partial^2(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t^2}\left\{\cos\Theta \pm \sqrt{-\sin^2\Theta + X}\right\}$$
$$+ \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\left\{-(-\omega_1 + \omega_\beta)\sin\Theta \pm \frac{1}{2}\frac{(-2)(-\omega_1 + \omega_\beta)\sin\Theta\cos\Theta + \frac{\partial X}{\partial t}}{\sqrt{-\sin^2\Theta + X}}\right\}$$
$$+ \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\left\{-(-\omega_1 + \omega_\beta)\sin\Theta \pm \frac{1}{2}\frac{(-2)(-\omega_1 + \omega_\beta)\sin\Theta\cos\Theta + \frac{\partial X}{\partial t}}{\sqrt{-\sin^2\Theta + X}}\right\} + (\boldsymbol{a}_2$$
$$+ \boldsymbol{b}\sqrt{X})\left\{-(-\omega_1 + \omega_\beta)^2\cos\Theta\right.$$
$$\pm\left(\frac{1}{2}\frac{(-2)(\cos^2\Theta - \sin^2\Theta)(-\omega_1 + \omega_\beta)^2 + \frac{\partial^2 X}{\partial t^2}}{\sqrt{-\sin^2\Theta + X}}\right.$$
$$\left.\left. + \frac{1}{2}\left(\frac{-1}{2}\right)\frac{\left[-2\sin\Theta\cos\Theta(-\omega_1 + \omega_\beta) + \frac{\partial X}{\partial t}\right]\left[-2\sin\Theta\cos\Theta(-\omega_1 + \omega_\beta) + \frac{\partial X}{\partial t}\right]}{\left(\sqrt[2]{-\sin^2\Theta + X}\right)^3}\right)\right\}$$

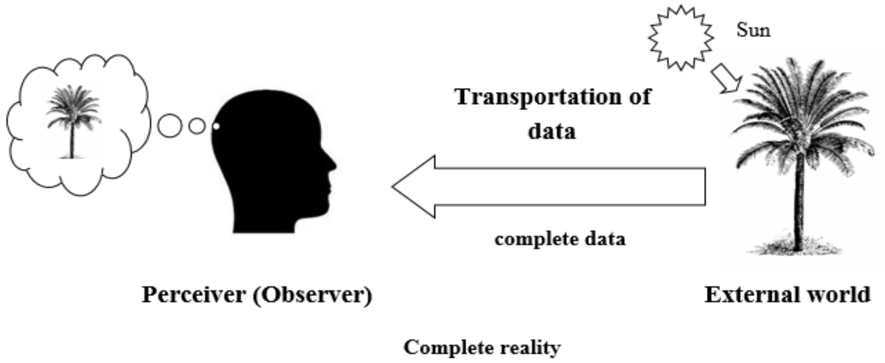
Empiricist approach
Toward a physical system

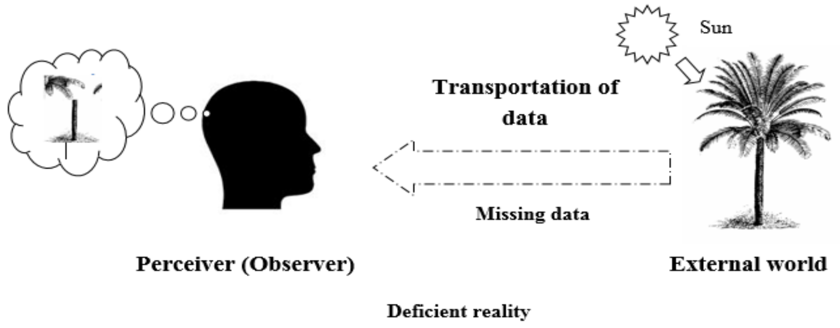
Partial observation and its effects

Mathematical external world

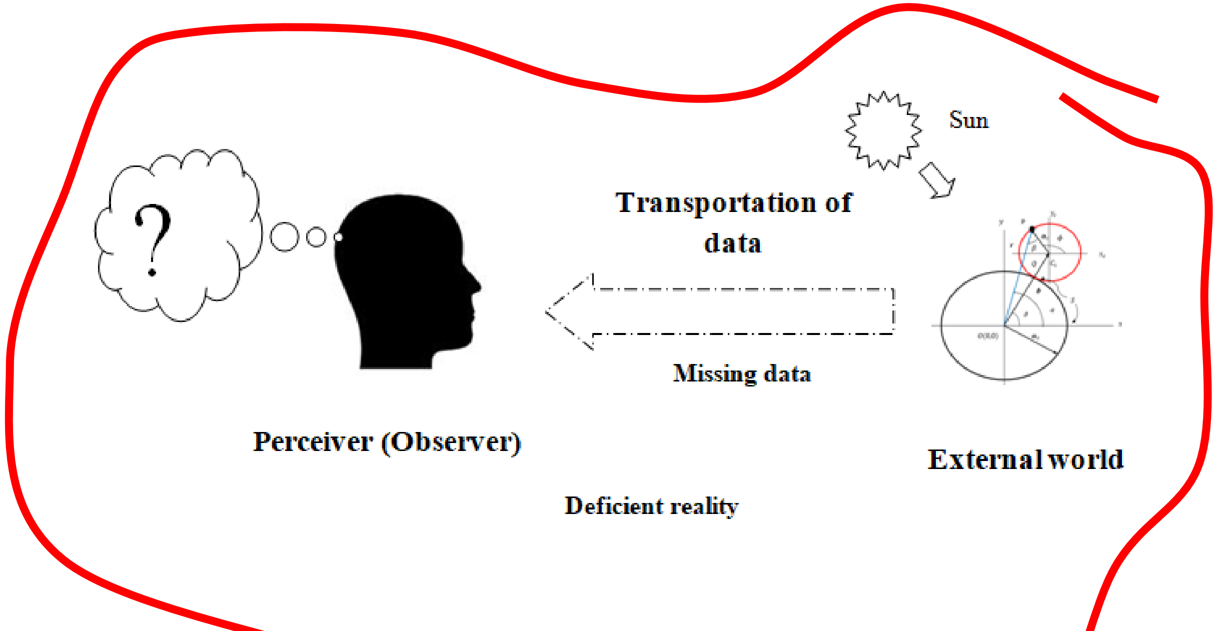
The independent external world is a hypothetical concept. The physical world is the observable world. Both worlds can be represented by mathematical models, but the first is a proposed mathematical model for a hypothetical external world, whereas the second is a mathematical model for an observable world. The experimental test for the second may prove that it is in agreement with the external world.







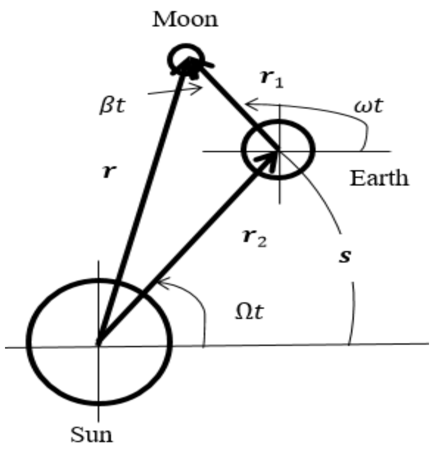
The deficient reality



The deficient reality

The propose of the external world and partial observation (Example)

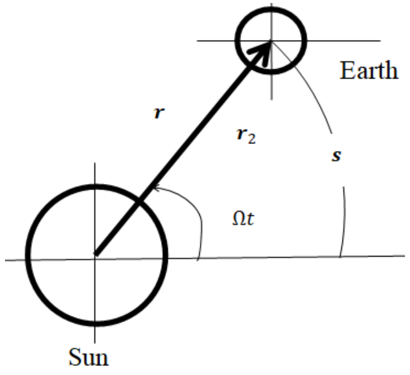
In case of the deformed system (emergent) the proposed external world depends on the conditions that leads to deformation and the observed model. This is an example:



External world, virtual (mathematical space)

$r_1 \ll \lambda \ll r_2$

$r_1 \approx 0$ and $r \approx r_2$



The system as observed (deformed), emergent

(Partial observation)

It is a process (using light for observation) of *physicalization* or to make the virtual system a physical system and can be observed under lab conditions.

The consequence of a strange theory

- The observer can explain the earth orbit, but can not see the distance between the earth and the moon. And can not see the moon as well. The moon looks as a shiny point on the earth.

- This case lead to build a strange theory to explain, the tide and the illumination of the earth during the night in certain periods and without the moon, but probably owing to a shiny spot rotate on the earth surface.

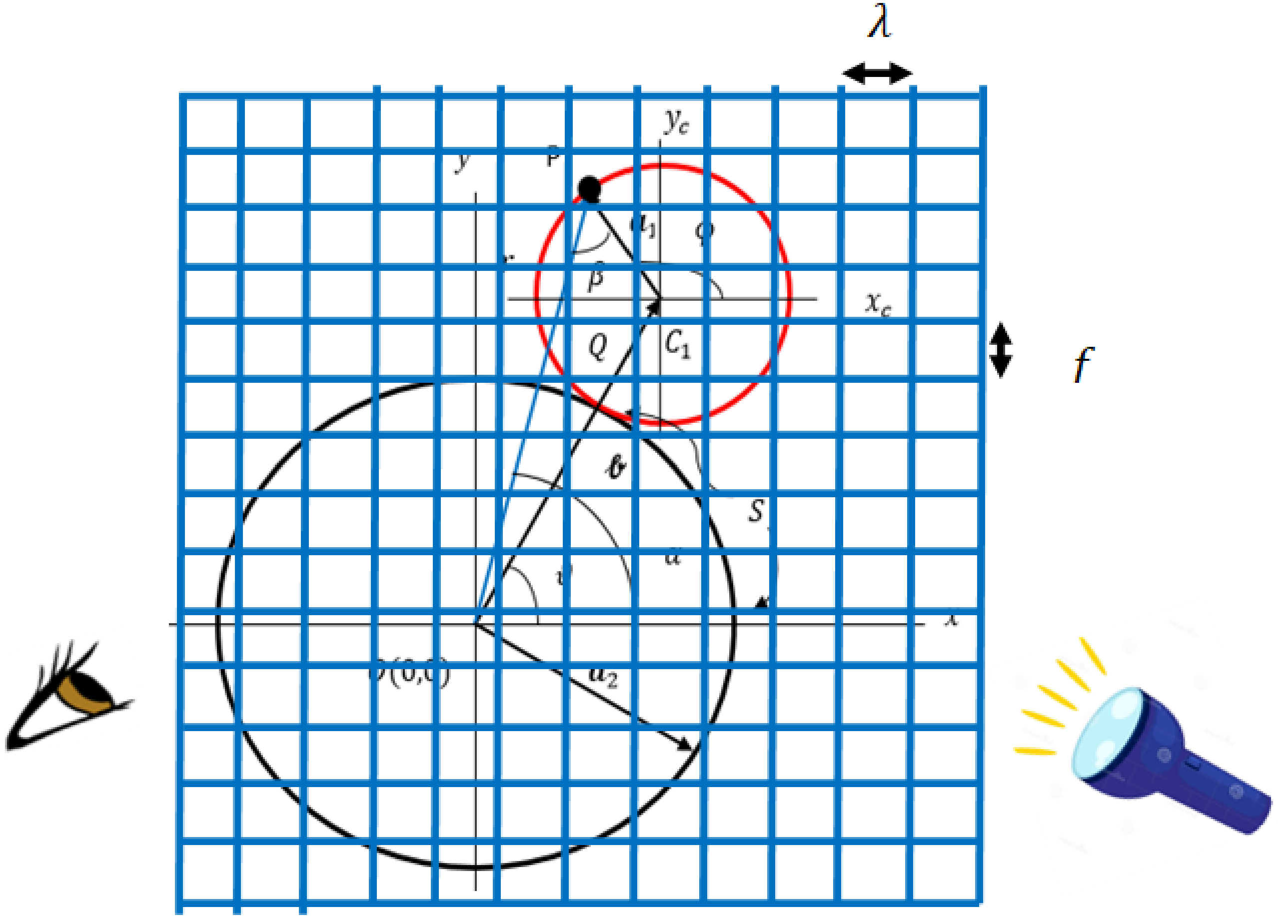
So the deformed system may lead to strange theories.

2-The optical resolution , transformation

Microscopic world

$$a_1 \ll d_\lambda \ll a_2,$$

$$\omega_1 \gg \omega_\lambda \gg \omega_2.$$



The spatial resolution () is the minimum linear distance between two distinguishable points

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1-The complex position vector

Substitution of $\boldsymbol{a}_1 \neq \boldsymbol{a}_{1m} = 0$, and $X_m = 0$

$\boldsymbol{b} \rightarrow \boldsymbol{a}_2 = \boldsymbol{a}_{2m}$. and $\omega_{\beta m} = \frac{\partial \beta}{\partial t} = 0$

$$\boldsymbol{r} = (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})\left\{\cos\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\}$$

in the position vector yields:

$$\boldsymbol{Z}\,(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m}\left\{\cos\left(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t\right) \pm \sqrt{-sin^2(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t)}\right\}.$$

or

$$\boldsymbol{Z}(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m} \exp \pm i\,(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t).$$

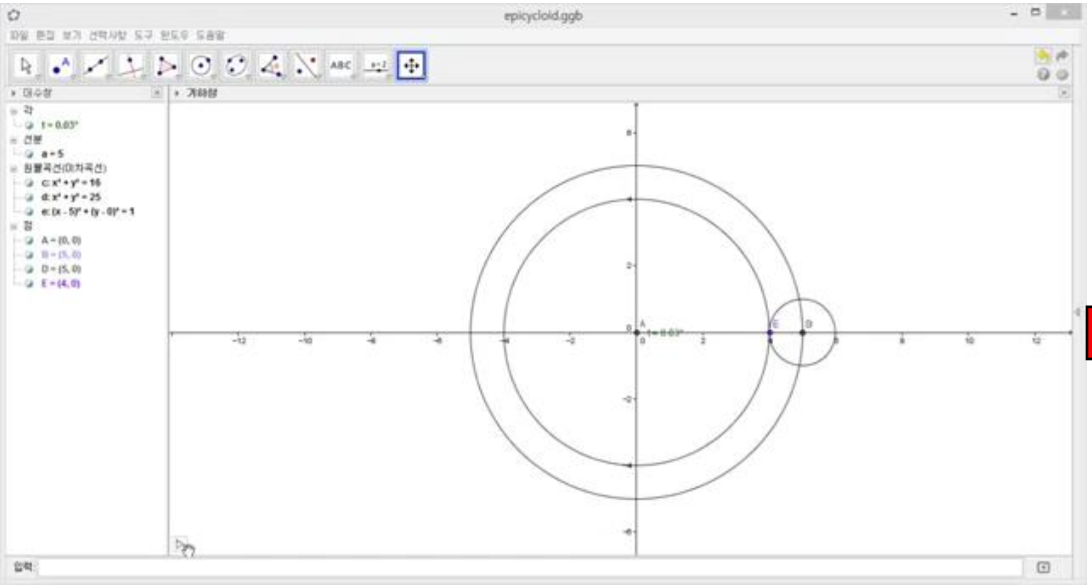
Sanduk, M., An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether, *International Journal of Quantum Foundations* 4 (2018) 173 – 198.

The physical system

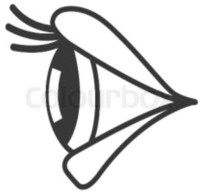
$$\boldsymbol{r} = \boldsymbol{b}\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-sin^2(\vartheta - \phi + \beta) + (\frac{a_1}{\boldsymbol{b}})^2}\right\}$$

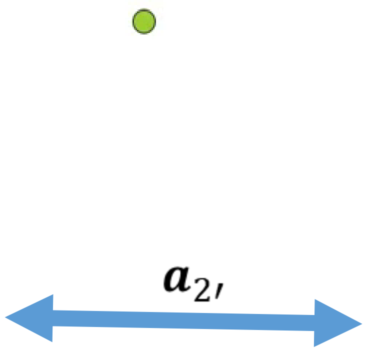
$$\boldsymbol{Z}(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m} \exp \pm i\,(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t).$$

Unlocalised point within the range of a_2,



External world





Observable world

2-The complex velocity equation

Under the partial observation and $\mathcal{Z}$ formulation, the velocity equation is transformed to complex velocity equation of this form:

$$
\begin{aligned}
&\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t} \\
&= \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{\mathscr{b}}\sqrt{X})}{\partial t}\Big\{ cos\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) \\
&\pm \sqrt{-sin^2\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) + X}\Big\} + (\boldsymbol{a}_2 \\
&+ \boldsymbol{\mathscr{b}}\sqrt{X})\Big\{ -(-\omega_1 + \omega_\beta) sin\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) \\
&\pm \left(\frac{1}{2}\right) \frac{(-2)(-\omega_1 + \omega_\beta)\, sin(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t) cos\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) + \frac{\partial X}{\partial t}}{\sqrt{-sin^2\big(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\big) + X}}\Big\}
\end{aligned}
$$

External world

$$
i\frac{\partial \mathcal{Z}}{\partial t} = (-i\, v\boldsymbol{A} \cdot \nabla + B\omega_{1m})\mathcal{Z}
$$

What is this equation?

Observable world

where $\boldsymbol{A} = -\hat{\boldsymbol{e}}_\vartheta$ $\quad B = \pm 1$

In relative to C2

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3-The complex acceleration equation

Under the partial observation and $\mathcal{Z}$ formulation, the acceleration equation is transformed to complex acceleration equation

$$
\begin{aligned}
&\frac{\partial^2 \boldsymbol{r}(r,t,X)}{\partial t^2} \\
&= \frac{\partial^2 (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t^2}\left\{\cos\Theta \pm \sqrt{-\sin^2\Theta + X}\right\} \\
&+ \frac{\partial (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\left\{-(-\omega_1+\omega_\beta)\sin\Theta \pm \frac{1}{2}\frac{(-2)(-\omega_1+\omega_\beta)\sin\Theta\cos\Theta + \frac{\partial X}{\partial t}}{\sqrt{-\sin^2\Theta + X}}\right\} \\
&+ \frac{\partial (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\left\{-(-\omega_1+\omega_\beta)\sin\Theta \pm \frac{1}{2}\frac{(-2)(-\omega_1+\omega_\beta)\sin\Theta\cos\Theta + \frac{\partial X}{\partial t}}{\sqrt{-\sin^2\Theta + X}}\right\} + (\boldsymbol{a}_2 \\
&+ \boldsymbol{b}\sqrt{X})\Bigg\{-(-\omega_1+\omega_\beta)^2\cos\Theta \\
&\pm\left(\frac{1}{2}\frac{(-2)(\cos^2\Theta - \sin^2\Theta)(-\omega_1+\omega_\beta)^2 + \frac{\partial^2 X}{\partial t^2}}{\sqrt{-\sin^2\Theta + X}} \right.\\
&\left. + \frac{1}{2}\left(\frac{-1}{2}\right)\frac{\left[-2\sin\Theta\cos\Theta(-\omega_1+\omega_\beta) + \frac{\partial X}{\partial t}\right]\left[-2\sin\Theta\cos\Theta(-\omega_1+\omega_\beta) + \frac{\partial X}{\partial t}\right]}{\left(\sqrt[2]{-\sin^2\Theta + X}\right)^3}\right)\Bigg\}
\end{aligned}
$$

External world

$$\frac{\partial^2 \mathcal{Z}}{\partial t^2} = (-v^2\,\nabla^2 + \omega_{1m}^2)\mathcal{Z}$$

Observable world

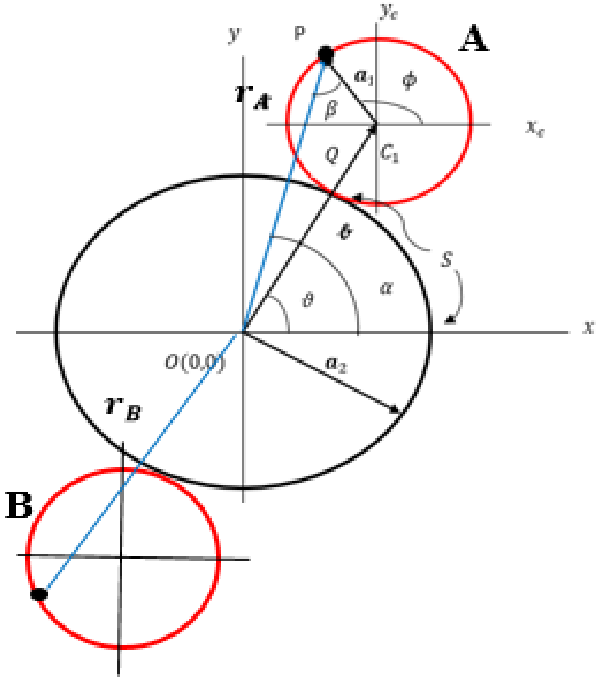
What is this equation of the lab observer?

4-High dimensional complex function

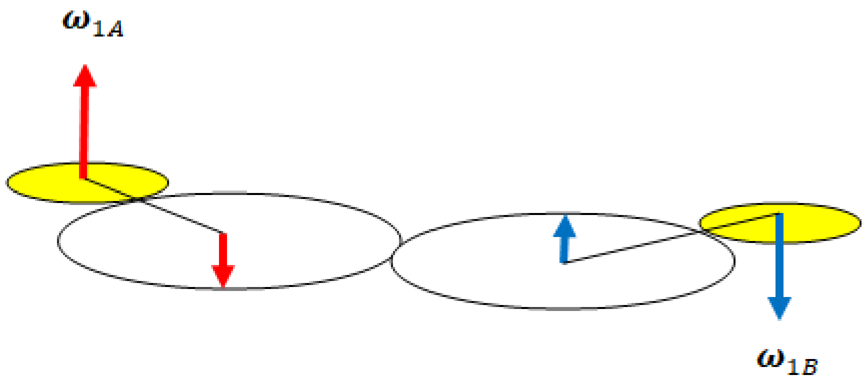
$$Z(r_A, r_B, \alpha_A, \alpha_B, t) = a\sqrt{2}exp^{\pm\frac{1}{2}i(k_{2A}\cdot s_A - \omega_{1A}t_A + k_{2B}\cdot s_B - \omega_{1B}t_B)}\sqrt{\cos\theta_c}$$

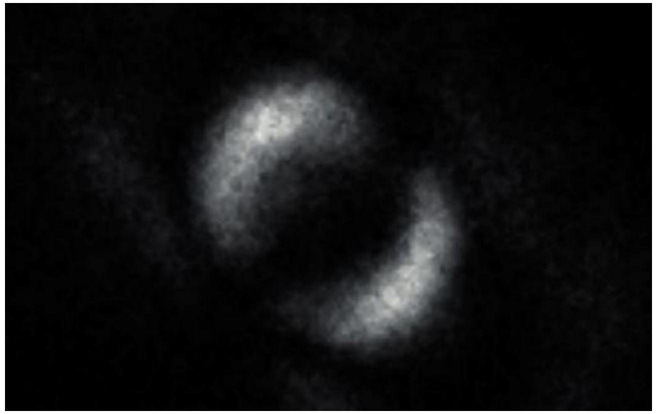
Four space dimensions

The high dimensional complex wave is arisen for two guided circles with one guiding circles under the effect of the partial observation. In this model, there is no need for a signal to be transmitted.



Sanduk, M., An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether, *International Journal of Quantum Foundations* 4 (2018) 173 - 198





$$Z(r_A, r_B, \alpha_A, \alpha_B, t) = a\sqrt{2}exp^{\pm\frac{1}{2}i(k_{2A}\cdot s_A - \omega_{1A}t_A + k_{2B}\cdot s_B - \omega_{1B}t_B)}\sqrt{\cos\theta_c}$$

Sanduk, M., An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether, *International Journal of Quantum Foundations* 4 (2018) 173 - 198

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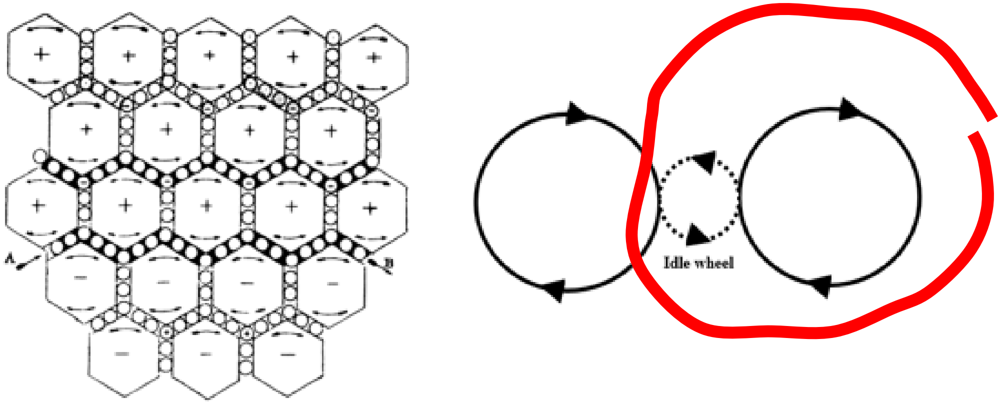
Comparisons table

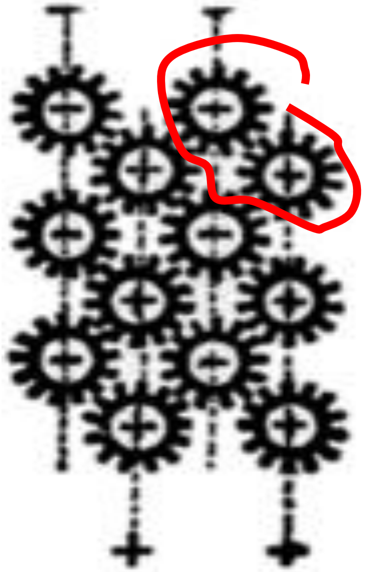
Conventional definition	Kinematic equations of relativistic quantum mechanics	Forms of the circles theory	Definition
Dirac wave function	$\psi_D = u_D \exp i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)$	$\boldsymbol{Z} = \boldsymbol{a}_{2m} \exp \pm i(\boldsymbol{k}_{2m}\cdot\boldsymbol{s} - \omega_{1m} t)$	*Z-complex vector*
Dirac equation	$i\frac{\partial\psi}{\partial t} = (-ic\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta\omega)\psi$	$i\frac{\partial\boldsymbol{Z}}{\partial t} = (-iv\boldsymbol{A}\cdot\boldsymbol{\nabla} + B\omega_{1m})\boldsymbol{Z}$	*Complex velocity equation*
The coefficients	$\boldsymbol{\alpha}$ and β	$\boldsymbol{A}$ and B	*Coefficient s*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_j + \boldsymbol{\alpha}_j\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}_\theta\cdot\boldsymbol{A}_\varphi + \boldsymbol{A}_\varphi\cdot\boldsymbol{A}_\theta = 0$	*Property*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i + \boldsymbol{\alpha}_i\boldsymbol{\alpha}_i = 2$	$\boldsymbol{A}_\theta\cdot\boldsymbol{A}_\theta + \boldsymbol{A}_\theta\cdot\boldsymbol{A}_\theta = 2$	*Property*
Property	$\alpha_i^2 = \beta^2 = 1$	$A^2 = B^2 = 1$	*Property*
Property	$\boldsymbol{\alpha}_i\beta + \beta\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}B + B\boldsymbol{A} = 0$	*Property*
Klein-Gordon equation	$\frac{\partial^2\psi}{\partial t^2} = [c^2\nabla^2 - \omega^2]\psi$	$\frac{\partial^2\boldsymbol{Z}}{\partial t^2} = [v^2\nabla^2 - \omega_{1m}{}^2]\boldsymbol{Z}$	*Complex accelerati on equation*

Sanduk, M., An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether, *International Journal of Quantum Foundations* 4 (2018) 173 - 198

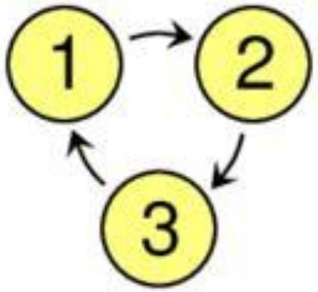
Cogwheel models



Maxwell's vortex æther model (1861).



Lodge's mechanical æther model (1889)



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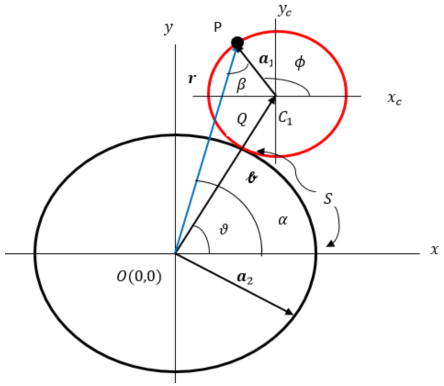

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Thank You