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Energy Conservation in A Relativistic Engine

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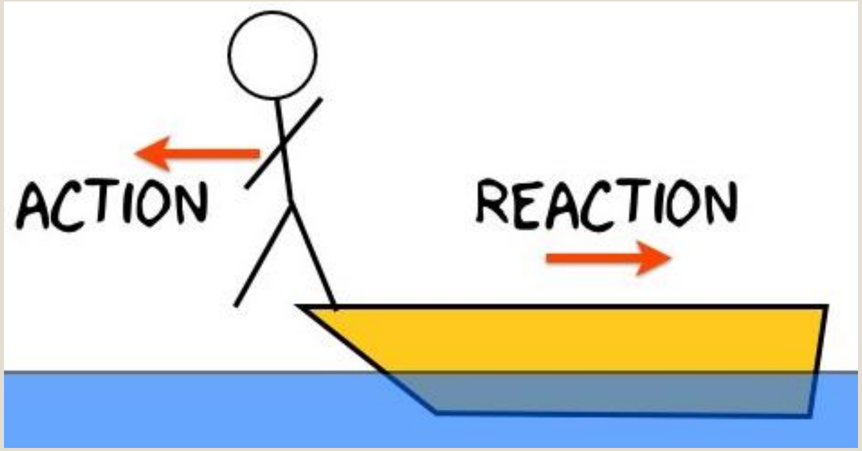
Contents

Retardation in Special Relativity

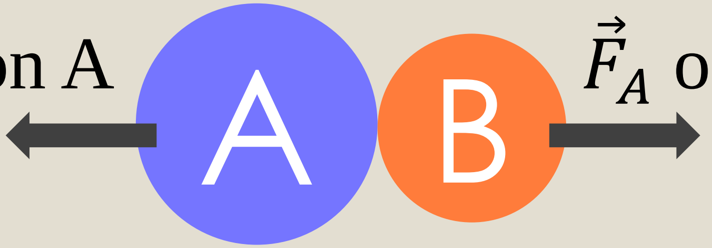
Maximum speed is finite: No physical object, message or field can travel faster than the speed of light in a vacuum.

Retardation: If someone at a distance $\boldsymbol{R}$ from me changes something then I may not know about it for at least a retardation time of $\boldsymbol{R/c}$.

Newton's Third Law



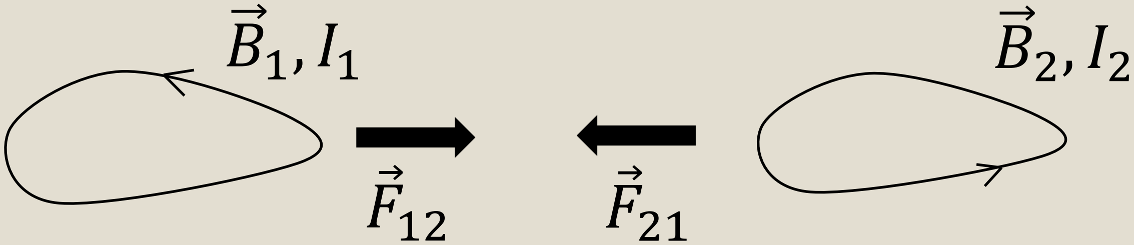
$\vec{F}_B$ on A



$\vec{F}_A$ on B

$$\vec{F}_B = -\vec{F}_A$$

The Case of Two Static Current Loops



Let us consider that two wires having segments of length $d\vec{l}_1$, $d\vec{l}_2$ located at $\vec{x}_1$, $\vec{x}_2$, respectively. The magneto-static force applied by current loop-2 on current loop-1 is given by:

$$\vec{F}_{12} = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{(d\vec{l}_1 \cdot d\vec{l}_2)\vec{x}_{12}}{|\vec{x}_{12}|^3}$$

Magneto-static force applied by current loop-1 on current loop-2:

$$\vec{F}_{21} = -\frac{\mu_0}{4\pi} I_2 I_1 \oint \oint \frac{(d\vec{l}_2 \cdot d\vec{l}_1)\vec{x}_{21}}{|\vec{x}_{21}|^3}$$

Since, $\vec{x}_{12} = \vec{x}_1 - \vec{x}_2 = -\vec{x}_{21}$, $|\vec{x}_{12}| = |\vec{x}_{21}| \equiv R$

We obtain $\vec{F}_{12} = -\vec{F}_{21}$

Total force on the two-loop system is null:

$$\vec{F}_T = \vec{F}_{12} + \vec{F}_{21} = 0$$

Time dependent case

Using Jefimenko's equation, the magnetic field can be written as:

$$\vec{B}(\vec{x}_2) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3 x_1 \frac{\vec{R}}{R^3} \times \left(\vec{J}_1(\vec{x}_1, t_{ret}) + \left(\frac{R}{c}\right) \partial_t \vec{J}_1(\vec{x}_1, t_{ret}) \right)$$

Quasi Static Approximation

$$t_{ret} \simeq t$$

Terms of order $\boldsymbol{\tau} = \boldsymbol{R}/\boldsymbol{c}$ are neglected.

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_1(t) I_2(t) \oint \oint \frac{(\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2)\vec{x}_{12}}{|\vec{x}_{12}|^3}$$

$$\vec{F}_{12} = -\frac{\mu_0}{4\pi} I_1(t) I_2(t) \oint \oint \frac{(\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2)\vec{x}_{12}}{|\vec{x}_{12}|^3}$$

Jefimenko, O. D., Electricity and Magnetism, Appleton-Century Crofts, New York (1966); 2nd edition, Electret Scientific, Star City, WV (1989).

Time dependent case

For finite time

The Taylor series expansion for current around t is written as:

$$I(t_{ret}) = I\left(t - \frac{R}{c}\right) = \sum_{n=0}^{\infty} \frac{I^{(n)}(t)}{n!}\left(-\frac{R}{c}\right)^n$$

Hence, the magnetic field can be written as follows:

$$\vec{B}(\vec{x}_2) = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{I_1^{(n)}(t)}{n!}\oint \vec{\nabla}_{\vec{x}_2} \times (\mathrm{d}\vec{l}_1 g_n(R))$$

where $g_n(R) = \frac{1}{R}(-\frac{R}{c})^n$

The force takes the form:

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_2(t) \sum_{n=0}^{\infty}\frac{I_1^{(n)}(t)}{n!}\left(-\frac{h}{c}\right)^n (1-n)\vec{K}_{21n}$$

where $\vec{K}_{21n} = \frac{1}{h^n}\oint\oint R^{n-3}\vec{R}(\mathrm{d}\vec{l}_2 \cdot \mathrm{d}\vec{l}_1) = -\vec{K}_{12n}$

where h is some characteristic distance between the coils

Time dependent case

The force due to coil 2 that acts on coil 1 is:

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1(t) \sum_{n=0}^{\infty} \frac{I_2^{(n)}(t)}{n!} \left(-\frac{h}{c}\right)^n (1-n)\vec{K}_{12n}$$

The total force on the system is thus:

$$\vec{F}_T = \vec{F}_{12} + \vec{F}_{21} = \frac{\mu_0}{4\pi} \sum_{n=0}^{\infty} \frac{(1-n)}{n!} \left(-\frac{h}{c}\right)^n \vec{K}_{12n} \left(I_1(t) I_2^{(n)}(t) - I_2(t) I_1^{(n)}(t)\right)$$

We note that there is no first-order contribution to the force. Hence the next contribution to the force after the quasi-static term is of second order.

$$\vec{F}_T = \frac{\mu_0}{4\pi} \sum_{n=2}^{\infty} \frac{(1-n)}{n!} \left(-\frac{h}{c}\right)^n \vec{K}_{12n} \left(I_1(t) I_2^{(n)}(t) - I_2(t) I_1^{(n)}(t)\right)$$

- The total force is not null.
- No zero (quasi static) nor first order contribution.
- The effect is second order in retardation time.

Relativistic Engine

- Our analysis led to the possibility of a relativistic engine for locomotive applications.
- The system is not composed of two material bodies but a material body and field.
- We will show that any momentum gained by the material part of the system is equal in magnitude and opposite in direction to the momentum gained by the electromagnetic field. Hence the total momentum of the system is conserved.

Momentum Generation

The total force:

$$\vec{F}_T \cong -\frac{\mu_0}{8\pi}\left(\frac{h}{c}\right)^2 \vec{K}_{122}\left(I_1(t)I_2^{(2)}(t) - I_2(t)I_1^{(2)}(t)\right)$$

Mechanical momentum:

$$\vec{P}_{mech} = \int_0^t \vec{F}_T(t')\mathrm{d}t' \cong -\frac{\mu_0}{8\pi}\left(\frac{h}{c}\right)^2 \vec{K}_{122}\left(I_1(t)I_2^{(1)}(t) - I_2(t)I_1^{(1)}(t)\right)$$

$$\vec{P}_{mech} \cong \frac{\mu_0}{8\pi} I_1^{(1)}(t) I_2 \left(\frac{h}{c}\right)^2 \vec{K}_{122}$$

Energy Generation

The kinetic mechanical energy associated with momentum is:

$$E_{mech} = \frac{\vec{P}^2_{mech}}{2M} = \frac{1}{2}\vec{P}_{mech} \cdot \vec{v} \propto \frac{1}{c^4}$$

were M is the mass of the relativistic engine and velocity of engine is as follows:

$$\vec{v} = \frac{\vec{P}_{mech}}{M} \propto \frac{1}{c^2}$$

We also note that the expression for mechanical energy is of order $1/c^4$, lower order corrections do not exist and higher order corrections are neglected.

Momentum Conservation

$$\vec{P}_{mech} = -\vec{P}_{field\,12} - \vec{P}_{field\,21}$$

If system 2 is magnetostatic then $E_2 = 0$. In such cases,

$$\vec{P}_{mech} = -\vec{P}_{field\,12}$$

Field momentum:

$$\vec{P}_{field\,12} \equiv \epsilon_0 \int \vec{E}_1 \times \vec{B}_2 \mathrm{d}^3 x$$

$$\vec{P}_{field\,12} = -\frac{\mu_0}{8\pi} I_2 \partial_t I_1(t) \frac{h^2}{c^2} \vec{K}_{122}$$

We observe that:

$$\vec{P}_{field\,12} = -\vec{P}_{mech}$$

Although Newton's Third Law does not hold exactly total momentum is conserved (total momentum = mechanical momentum + field momentum).

Energy Conservation

$$\frac{dE_{mech}}{dt} + \frac{dE_{field}}{dt} = -\oint_S \vec{S}_p \cdot \hat{n} da$$

$$E_{field} \equiv \int e_{field} d^3x = \frac{\epsilon_0}{2} \int \left(\vec{E}^2 + c^2 \vec{B}^2 \right) d^3x$$

$$\vec{S}_p = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Energy Conservation

Consider two sub-systems denoted system 1 and system 2 which are far apart such that their interaction is negligible.

$$\int d^3x \vec{J}_1 \cdot \vec{E}_1 = -\frac{dE_{field\ 1}}{dt} - \oint_S \vec{S}_{p\ 1} \cdot \hat{n} da.$$

$$\int d^3x \vec{J}_2 \cdot \vec{E}_2 = -\frac{dE_{field\ 2}}{dt} - \oint_S \vec{S}_{p\ 2} \cdot \hat{n} da.$$

Next we will put the two loops closer together such that they may interact but without modifying the charge and the current densities of each of the subsystems. The total fields of the combined system are:

$$\vec{E} = \vec{E}_1 + \vec{E}_2, \qquad \vec{B} = \vec{B}_1 + \vec{B}_2$$

Field Energy

$$E_{field} \equiv \frac{\epsilon_0}{2} \int \left(\vec{E}^2 + c^2 \vec{B}^2 \right) d^3x = E_{field\ 1} + E_{field\ 2} + E_{field\ 12}$$

$$\begin{aligned} E_{field\ 1} &\equiv E_{Efield\ 1} + E_{Mfield\ 1} \\ E_{field\ 2} &\equiv E_{Efield\ 2} + E_{Mfield\ 2} \end{aligned}$$

$$\begin{aligned} E_{field\ 12} &\equiv E_{Efield\ 12} + E_{Mfield\ 12} \\ &\equiv \epsilon_0 \int \left(\vec{E}_1 \cdot \vec{E}_2 + c^2 \vec{B}_1 \cdot \vec{B}_2 \right) d^3x \end{aligned}$$

$$Power_{12} = -\frac{dE_{field\ 12}}{dt} - \oint_S \vec{S}_{p\ 12} \cdot \hat{n} da$$

System of two current loops

We need to consider two types of time dependence. One is due to the intrinsic time dependence of the current flowing through the loop and another is due to its movement as part of the relativistic engine. Thus the current density can be written as:

$$\vec{J}(\vec{x},t) = \vec{J}'(\vec{x} - \vec{x}_c(t), t)$$

$\vec{J}'(\vec{x},t)$ is current density in moving frame of relativistic engine

$\frac{d\vec{x}_c(t)}{dt} = \vec{v}(t)$ is the velocity of the relativistic engine.

System of two current loops

The vector potential is given as:

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}',t_{ret})}{R}, \qquad \vec{R} \equiv \vec{x} - \vec{x}', \qquad t_{ret} \equiv t - \frac{R}{c}$$

$$\begin{aligned}\vec{A}(\vec{x},t) &= \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\int d^3x' \frac{1}{R}(-\frac{R}{c})^n \frac{d^n}{dt^n}\vec{J}(\vec{x}',t) \\ &= \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{dt^n}\int d^3x' \frac{1}{R}(-\frac{R}{c})^n \vec{J}'(\vec{x}' - \vec{x}_c(t),t),\end{aligned}$$

System of two current loops

Let us introduce a comoving integration variable:

$$\vec{\tilde{x}} = \vec{x}' - \vec{x}_c(t) \qquad R(t) = |\vec{x}' - \vec{x}| = |\vec{\tilde{x}} + \vec{x}_c(t) - \vec{x}|$$

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{dt^n}\int d^3\tilde{x}\frac{1}{R(t)}(-\frac{R(t)}{c})^n \vec{J}'(\vec{\tilde{x}},t)$$

For a thin and uniform current loop this can be written as:

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{d^n}{dt^n}\left[I(t)\oint d\vec{\tilde{l}}\frac{1}{R(t)}(-\frac{R(t)}{c})^n\right]$$

The zeroth order approximation takes the form:

$$\vec{A}(\vec{x},t) \simeq \frac{\mu_0}{4\pi}I(t)\oint d\vec{\tilde{l}}\frac{1}{R(t)}$$

System of two current loops

Let us define:

$$\vec{A}^{(n)}(\vec{x},t) = \frac{\mu_0}{4\pi n!}\frac{d^n}{dt^n}\left[I(t)\oint d\tilde{\vec{l}}\frac{1}{R(t)}\left(-\frac{R(t)}{c}\right)^n\right]$$

$$= (-1)^n\frac{\mu_0}{4\pi n! c^n}\frac{d^n}{dt^n}\left[I(t)\oint d\tilde{\vec{l}}R(t)^{n-1}\right]$$

We may write: $$\vec{A}(\vec{x},t) = \sum_{n=0}^{\infty}\vec{A}^{(n)}(\vec{x},t)$$

$$\vec{E}^{(n)} \equiv -\partial_t \vec{A}^{(n)}$$

$$\vec{B}^{[n]} = \vec{\nabla}\times\vec{A}^{[n]}$$

We will use the above expressions to analyzed the energy transformation order by order starting from n = 0 and up to n = 4.

n = 0

$$Power'^{[0]}_{12} = -\frac{dE'^{[0]}_{field\ 12}}{dt} - \oint_S \vec{S}'^{[0]}_{p\ 12} \cdot \hat{n} da$$

$$Power^{[0]}_{12} = \frac{\mu_0}{16\pi^2} Power'^{[0]}_{12} = -\partial_t I_1(t) I_2 M^{[0]}_{12}$$

Mutual inductance: $M^{[0]}_{12} \equiv \frac{\mu_0}{4\pi} \oint d\vec{l_1} \cdot \oint d\vec{l_2} \frac{1}{|\vec{x}_1 - \vec{x}_2|}$

We notice that Power may be positive or negative according to the relative position of the current loops and current directions.

Field energy for n = 0

$$E'^{[0]}_{field\ 12} = \int \vec{B}'^{[0]}_1 \cdot \vec{B}'^{[0]}_2 d^3x$$

$$\vec{B}'^{[0]}(\vec{x},t) = -I(t) \oint d\tilde{\vec{l}} \times \vec{\nabla} \frac{1}{R(t)} = I(t) \oint d\tilde{\vec{l}} \times \frac{\vec{R}(t)}{R^3(t)}$$

$$E^{[0]}_{field\ 12} = \frac{\mu_0}{(4\pi)^2} E'^{[0]}_{field\ 12} = I_1(t) I_2 M^{[0]}_{12}$$

Poynting vector for n = 0

$$\vec{S}'^{[0]}_{p\ 12} = \vec{E}'^{[0]}_1 \times \vec{B}'^{[0]}_2 + \vec{E}'^{[0]}_2 \times \vec{B}'^{[0]}_1$$

As is null $\vec{E}'^{[0]}_2$, hence $\vec{S}'^{[0]}_{p\ 12} = \vec{E}'^{[0]}_1 \times \vec{B}'^{[0]}_2$

$$\oint_S \vec{S}'^{[0]}_{p\ 12} \cdot \hat{n} da = \lim_{r\to\infty} \oint_S (\vec{S}'^{[0]}_{p\ 12} \cdot \hat{r}) r^2 d\Omega = 0$$

There is no Poynting vector contribution to the energy balance. Thus is the quasi static approximation there is no radiation losses as expected.

n = 0

Mechanical work invested or extracted in the system results in increase or decrease in the field energy accordingly. The power related to the mechanical work is:

$$Power_{12}^{[0]} = -\partial_t I_1(t) I_2 M_{12}^{[0]}$$

and this is equal to minus the derivative of the field energy:

$$E_{field\ 12}^{[0]} = I_1(t) I_2 M_{12}^{[0]}$$

We underline that those contributions are not related the relativistic engine effect as non of the terms depends on the engine velocity.

n = 1

$$Power'^{[1]}_{12} = -\frac{dE'^{[1]}_{field\ 12}}{dt} - \oint_S \vec{S}'^{[1]}_{p\ 12} \cdot \hat{n} da$$

$$Power'^{[1]}_{12} = 4\pi \left(I_1(t) \oint d\tilde{\vec{l}_1} \cdot \vec{E}'^{[1]}_2(\vec{x}_1) + I_2(t) \oint d\tilde{\vec{l}_2} \cdot \vec{E}'^{[1]}_1(\vec{x}_2) \right)$$

The field $\vec{E}'^{[1]}$ can only come from the vector potential $\vec{A}'^{[1]}$. As $\vec{A}'^{(1)} = 0$, it follows that $\vec{A}'^{[1]} = 0$. Hence:

$$\vec{E}'^{[1]} = 0 \qquad \vec{B}'^{[1]}(\vec{x}, t) = 0$$

$$Power'^{[1]}_{12} = 0 \qquad E'^{[1]}_{field\ 12} = 0$$

Hence no mechanical work is invested nor extracted for a relativistic engine in the first order of 1/c. There is also no contribution from first order terms in 1/c to the field energy of a relativistic engine neither.

n = 2

$$Power'^{[2]}_{12} = -\frac{dE'^{[2]}_{field\ 12}}{dt} - \oint_S \vec{S}'^{[2]}_{p\ 12} \cdot \hat{n} da$$

$$\vec{A}'^{[2]}(\vec{x}, t) \equiv \frac{1}{2c^2} \frac{d^2 I(t)}{dt^2} \oint d\tilde{\vec{l}} R(t)$$

$$\vec{E}'^{[2]}_2 = -I_2 \oint d\tilde{\vec{l}_2} \quad \frac{\vec{v} \cdot \vec{R}_2(t)}{R_2^3(t)}$$

$$Power^{[2]}_{12} = \frac{\mu_0}{16\pi^2} Power'^{[2]}_{12} = -\frac{\mu_0}{8\pi c^2} \frac{d^3 I_1(t)}{dt^3} I_2 \oint \oint d\vec{l_2} \cdot d\vec{l_2} R_{12}$$

This term has nothing to do with the relativistic engine effect as it is completely independent of the mass of the engine.

Field energy for n = 2

$$E'^{[2]}_{field\ 12} = \int \left(\frac{1}{c^2} \vec{E}'^{[0]}_1 \cdot \vec{E}'^{[0]}_2 + \vec{B}'^{[0]}_1 \cdot \vec{B}'^{[2]}_2 + \vec{B}'^{[2]}_1 \cdot \vec{B}'^{[0]}_2 \right) d^3x.$$

Zeroth order electric field for static current second coil is null, hence:

$$E'^{[2]}_{field\ 12} = \int \left(\vec{B}'^{[0]}_1 \cdot \vec{B}'^{[2]}_2 + \vec{B}'^{[2]}_1 \cdot \vec{B}'^{[0]}_2 \right) d^3x$$

$$\vec{B}'^{[2]}(\vec{x},t) = -\frac{1}{2c^2} \frac{d^2 I(t)}{dt^2} \oint d\tilde{\vec{l}} \times \vec{\nabla} R(t) = -\frac{1}{2c^2} \frac{d^2 I(t)}{dt^2} \oint d\tilde{\vec{l}} \times \frac{\vec{R}(t)}{R(t)}$$

Hence for a static coil:

$$E'^{[2]}_{field\ 12} = \int \vec{B}'^{[2]}_1 \cdot \vec{B}'^{[0]}_2 d^3x$$

$$E^{[2]}_{field\ 12} = \frac{\mu_0}{(4\pi)^2} E'^{[2]}_{field\ 12} = \frac{\mu_0}{8\pi c^2} \frac{d^2 I_1(t)}{dt^2} I_2 \oint \oint (d\vec{l_1} \cdot d\vec{l_2}) R_{12}$$

Field energy for n = 2

For a phasor current with frequency ω, we obtain a second order correction to the mutual inductance :

$$M_{12}^{[2]} \equiv -\frac{\mu_0 \omega^2}{8\pi c^2} \oint d\vec{l}_1 \cdot \oint d\vec{l}_2 R_{12}$$

$$E_{field\ 12}^{[2]} = I_1(t) I_2 M_{12}^{[2]}$$

We stress that this term is not related to the relativistic engine effect and will exist even for an engine of "infinite" mass.

$$\oint_S \vec{S}'^{[2]}_{p\ 12} \cdot \hat{n} da = \lim_{r\to\infty} \oint_S (\vec{S}'^{[2]}_{p\ 12} \cdot \hat{r}) r^2 d\Omega = 0$$

- There is no Poynting vector contribution to the energy balance.
- The mechanical work is balanced by the field energy loss exactly.

n = 3

$$Power'^{[3]}_{12} = -\frac{dE'^{[3]}_{field\ 12}}{dt} - \oint_S \vec{S}'^{[3]}_{p\ 12} \cdot \hat{n} da$$

$$Power'^{[3]}_{12} = 4\pi \left(I_1(t) \oint d\tilde{\vec{l_1}} \cdot \vec{E}'^{[3]}_2(\vec{x}_1) + I_2(t) \oint d\tilde{\vec{l_2}} \cdot \vec{E}'^{[3]}_1(\vec{x}_2) \right)$$

$$\vec{E}'^{[3]} = \frac{1}{6c^3} \frac{d^4 I(t)}{dt^4} \oint d\vec{l} R^2$$

$$Power^{[3]}_{12} = \frac{\mu_0}{16\pi^2} Power'^{[3]}_{12} = \frac{\mu_0}{24\pi c^3} \frac{d^4 I_1(t)}{dt^4} I_2 \oint \oint d\vec{l_1} \cdot d\vec{l_2} R^2_{12}$$

This term has nothing to do with the relativistic engine effect as it is completely independent of the mass of the engine.

Field energy for n = 3

$$E'^{[3]}_{field\ 12} = \int \left(\vec{B}'^{[0]}_1 \cdot \vec{B}'^{[3]}_2 + \vec{B}'^{[3]}_1 \cdot \vec{B}'^{[0]}_2\right) d^3x,$$

Hence for a static coil: $\vec{B}'^{[3]}_2(\vec{x}, t) = 0$

$$E'^{[3]}_{field\ 12} = \int \vec{B}'^{[3]}_1 \cdot \vec{B}'^{[0]}_2 d^3x$$

$$\begin{aligned} E'^{[3]}_{field\ 12} &= -\frac{2\pi}{3c^3}\frac{d^3 I_1(t)}{dt^3} I_2[\oint\oint(d\vec{l_1} \cdot d\vec{l_2})R^2_{12} - \frac{2}{3}\oint\oint(d\vec{l_1} \cdot d\vec{l_2})(\vec{x}_1 \cdot \vec{x}_2) \\ &+ \frac{2}{3}\oint\oint(d\vec{l_1} \cdot \vec{x}_2)(d\vec{l_2} \cdot \vec{x}_1)]. \end{aligned}$$

We will dissect the above expression into volume and surface contributions such that:

$$E'^{[3]}_{field\ 12} = E'^{[3]}_{fieldV\ 12} + E'^{[3]}_{fieldS\ 12}$$

Field energy for n = 3

$$E^{[3]}_{fieldV\ 12} = \frac{\mu_0}{(4\pi)^2} E'^{[3]}_{fieldV\ 12} = -\frac{\mu_0}{24\pi c^3} \frac{d^3 I_1(t)}{dt^3} I_2 \oint \oint (d\vec{l_1} \cdot d\vec{l_2}) R^2_{12}$$

We obtain a second order correction to the mutual inductance of the form:

$$M^{[3]}_{12} \equiv \frac{j\mu_0\omega^3}{24\pi c^3} \oint d\vec{l_1} \cdot \oint d\vec{l_2} R^2_{12}$$

$$E^{[3]}_{field\ 12} = I_1(t) I_2 M^{[3]}_{12}$$

This term is not related to the relativistic engine effect. We have unbalanced surface terms with field energy:

$$E^{[3]}_{fieldS\ 12} = \frac{\mu_0}{(4\pi)^2} E'^{[3]}_{fieldS\ 12} = \frac{\mu_0}{36\pi c^3} \frac{d^3 I_1(t)}{dt^3} I_2 \left[\oint \oint (d\vec{l_1} \cdot d\vec{l_2})(\vec{x}_1 \cdot \vec{x}_2) - \oint \oint (d\vec{l_1} \cdot \vec{x}_2)(d\vec{l_2} \cdot \vec{x}_1) \right]$$

Field energy for n = 3

$$E^{[3]}_{fieldS\ 12} = \frac{\mu_0}{9\pi c^3} I_2 \frac{d^3 I_1(t)}{dt^3} \vec{A}r_1 \cdot \vec{A}r_2$$

Hence orthogonal current loops will generate a null surface field contribution. Finally we shall study the Poynting vector:

$$\vec{S}'^{[3]}_{p\ 12} = \vec{E}'^{[0]}_1 \times \vec{B}'^{[3]}_2 + \vec{E}'^{[3]}_1 \times \vec{B}'^{[0]}_2 + \vec{E}'^{[0]}_2 \times \vec{B}'^{[3]}_1 + \vec{E}'^{[3]}_2 \times \vec{B}'^{[0]}_1$$

$$\oint_S \vec{S}^{[3]}_{p\ 12} \cdot \hat{n} da = \frac{\mu_0}{(4\pi)^2} \oint_S \vec{S}'^{[3]}_{p\ 12} \cdot \hat{n} da = -\frac{\mu_0}{9\pi c^3} I_2 \frac{d^4 I_1(t)}{dt^4} \vec{A}r_1 \cdot \vec{A}r_2$$

- The mechanical work is balanced by the change in volume field energy.
- The Poynting radiation flux balances that change in surface field energy.

n = 4

$$Power'^{[4]}_{12} = -\frac{dE'^{[4]}_{field\ 12}}{dt} - \oint_S \vec{S}'^{[4]}_{p\ 12} \cdot \hat{n} da$$

Power can be calculated as:

$$Power'^{[4]}_{12} = 4\pi \left(I_1(t) \oint d\tilde{\vec{l_1}} \cdot \vec{E}'^{[4]}_2(\vec{x}_1) + I_2(t) \oint d\tilde{\vec{l_2}} \cdot \vec{E}'^{[4]}_1(\vec{x}_2) \right)$$

Total mechanical work done in the fourth order can be calculated as:

$$Power^{[4]}_{12} = Power^{[4]}_{12a} + Power^{[4]}_{12b}$$

$$Power'^{[4]}_{12a} = \frac{2\pi}{c^2} \frac{d^2 I_1(t)}{dt^2} I_2 \oint d\tilde{\vec{l_2}} \cdot \oint d\tilde{\vec{l_1}} \hat{R}_{21} \cdot \vec{v}$$

n = 4

$$Power^{[4]}_{12a} = \frac{\mu_0}{16\pi^2} Power'^{[4]}_{12a} = \frac{\mu_0 h^2}{8\pi c^2} \frac{d^2 I_1(t)}{dt^4} I_2 \vec{K}_{122} \cdot \vec{v}.$$

$$Power^{[4]}_{12a} = \frac{d\vec{P}_{mech}}{dt} \cdot \vec{v}.$$

which is exactly the amount of mechanical power needed to drive the relativistic engine. Unfortunately more power is needed to drive the currents through the coils as will be demonstrated below.

$$Power^{[4]}_{12b} = \frac{\mu_0}{16\pi^2} Power'^{[4]}_{12b}$$

$$Power^{[4]}_{12b} = -\frac{\mu_0}{96\pi c^4} \frac{d^5 I_1(t)}{dt^5} I_2 \oint \oint d(\vec{l_1} \cdot d\vec{l_2}) R^3_{21} + 5\vec{P}_{mech} \cdot \frac{d\vec{v}}{dt}$$

n = 4

Total mechanical work done:

$$\begin{aligned} Power_{12}^{[4]} &= Power_{12a}^{[4]} + Power_{12b}^{[4]} \\ &= -\frac{\mu_0}{96\pi c^2}\frac{d^5 I_1(t)}{dt^5} I_2 \oint\oint (d\vec{l_1}\cdot d\vec{l_2}) R_{21}^3 + 6\vec{P}_{mech}\cdot\frac{d\vec{v}}{dt} \\ &= -\frac{\mu_0}{96\pi c^2}\frac{d^5 I_1(t)}{dt^5} I_2 \oint\oint (d\vec{l_1}\cdot d\vec{l_2}) R_{21}^3 + 6\frac{dE_{mech}}{dt}. \end{aligned}$$

- The mechanical work to fourth order in 1/c has two parts.
- One is clearly not related the relativistic engine effect and the other which clearly is.
- The mechanical power needed to operate the relativistic engine it is six times greater then the change in kinetic energy of the engine itself.
- The rest of the power is invested in driving the currents through the coils.

Field energy for n = 4

Field energy for fourth order term in 1/c is expressed as:

$$E'^{[4]}_{field\ 12} = \int \left(\frac{1}{c^2} \vec{E}'^{[0]}_1 \cdot \vec{E}'^{[2]}_2 + \vec{B}'^{[0]}_1 \cdot \vec{B}'^{[4]}_2 + \vec{B}'^{[4]}_1 \cdot \vec{B}'^{[0]}_2 \right) d^3x$$

The field energy can be partitioned to electric field and magnetic field contributions:

$$\begin{aligned} E'^{[4]}_{field\ 12} &= E'^{[4]}_{Efield\ 12} + E'^{[4]}_{Mfield\ 12} \\ E'^{[4]}_{Efield\ 12} &= \frac{1}{c^2} \int \vec{E}'^{[0]}_1 \cdot \vec{E}'^{[2]}_2 \\ E'^{[4]}_{Mfield\ 12} &= \int \left(\vec{B}'^{[0]}_1 \cdot \vec{B}'^{[4]}_2 + \vec{B}'^{[4]}_1 \cdot \vec{B}'^{[0]}_2 \right) d^3x \end{aligned}$$

Field energy for n = 4

Electric part of field energy is calculated as:

$$E'^{[4]}_{Efield\ 12} = \frac{I_2}{c^2}\frac{dI_1(t)}{dt}\oint\oint(d\vec{l_1}\cdot d\vec{l_2})\vec{v}\cdot\int d^3x\frac{\vec{R}_2}{R_2^2 R_1}$$

$$E^{[4]}_{Efield\ 12} = \frac{\mu_0}{(4\pi)^2}E'^{[4]}_{Efield\ 12} = -\frac{\mu_0}{8\pi c^2}\frac{dI_1(t)}{dt}I_2\vec{v}\cdot\vec{K}_{122}$$

$$E^{[4]}_{Efield\ 12} = -\vec{v}\cdot\vec{P}_{mech} = -2E_{mech}$$

Field energy for n = 4

Magnetic part of field energy

Fourth order correction of the magnetic field can be calculated according to:

$$\begin{aligned}\vec{B}'^{[4]}(\vec{x},t) &= -\frac{I(t)}{2c^2}\oint \vec{\nabla}(\hat{R}\cdot\frac{d\vec{v}}{dt})\times d\tilde{\vec{l}} - \frac{1}{c^2}\frac{dI(t)}{dt}\oint \vec{\nabla}(\hat{R}\cdot\vec{v})\times d\tilde{\vec{l}} \\ &+ \frac{1}{24c^4}\frac{d^4I(t)}{dt^4}\oint \vec{\nabla}R^3(t)\times d\tilde{\vec{l}}.\end{aligned}$$

Hence for a static current:

$$\vec{B}_2'^{[4]}(\vec{x},t) = -\frac{I_2}{2c^2}\oint \vec{\nabla}(\hat{R}_2\cdot\frac{d\vec{v}}{dt})\times d\tilde{\vec{l}_2}$$

Field energy for n = 4

We can label the different terms of magnetic field of fourth order:

$$\begin{aligned}
\vec{B}'^{[4]}(\vec{x},t) &= \vec{B}_a'^{[4]} + \vec{B}_b'^{[4]} + \vec{B}_c'^{[4]} \\
\vec{B}_a'^{[4]} &= -\frac{I(t)}{2c^2}\oint \vec{\nabla}(\hat{R}\cdot\frac{d\vec{v}}{dt})\times d\tilde{\vec{l}} \\
\vec{B}_b'^{[4]} &= -\frac{1}{c^2}\frac{dI(t)}{dt}\oint \vec{\nabla}(\hat{R}\cdot\vec{v})\times d\tilde{\vec{l}} \\
\vec{B}_c'^{[4]} &= \frac{1}{24c^4}\frac{d^4I(t)}{dt^4}\oint \vec{\nabla}R^3(t)\times d\tilde{\vec{l}}.
\end{aligned}$$

The magnetic energy can also be partitioned:

$$\begin{aligned}
E'^{[4]}_{M field\ 12} &= E'^{[4]}_{M field\ 120} + E'^{[4]}_{M field\ 12a} + E'^{[4]}_{M field\ 12b} + E'^{[4]}_{M field\ 12c} \\
E'^{[4]}_{M field\ 120} &= \int \vec{B}_1'^{[0]}\cdot\vec{B}_2'^{[4]} d^3x \\
E'^{[4]}_{M field\ 12a} &= \int \vec{B}_{1a}'^{[4]}\cdot\vec{B}_2'^{[0]} d^3x \\
E'^{[4]}_{M field\ 12b} &= \int \vec{B}_{1b}'^{[4]}\cdot\vec{B}_2'^{[0]} d^3x \\
E'^{[4]}_{M field\ 12c} &= \int \vec{B}_{1c}'^{[4]}\cdot\vec{B}_2'^{[0]} d^3x.
\end{aligned}$$

Field energy for n = 4

$$E'^{[4]}_{M field\ 120} = 2\pi I_1(t) I_2 \frac{h^2}{c^2} \vec{K}_{122} \cdot \frac{d\vec{v}}{dt}$$

$$E'^{[4]}_{M field\ 12a} = 2\pi I_1(t) I_2 \frac{h^2}{c^2} \vec{K}_{212} \cdot \frac{d\vec{v}}{dt}$$

As $\vec{K}_{212} = -\vec{K}_{122}$, hence $E'^{[4]}_{M field\ 12a} = -E'^{[4]}_{M field\ 120}$

We arrive at: $E'^{[4]}_{M field\ 12} = E'^{[4]}_{M field\ 12b} + E'^{[4]}_{M field\ 12c}$

Field energy for n = 4

$$E'^{[4]}_{M\,field\ \ 12b} = 4\pi \frac{I_2}{c^2} \frac{dI_1(t)}{dt} \oint \oint \left(d\vec{l_1} \cdot d\vec{l_2} \right) \hat{R}_{12} \cdot \vec{v}.$$

$$E^{[4]}_{M\,field\ \ 12b} = \frac{\mu_0}{(4\pi)^2} E'^{[4]}_{M\,field\ \ 12b} = -\frac{\mu_0}{4\pi} I_2 \frac{dI_1(t)}{dt} \frac{h^2}{c^2} \vec{K}_{122} \cdot \vec{v}.$$

$$E^{[4]}_{M\,field\ \ 12b} = -2\vec{P}_{mech} \cdot \vec{v} = -4E_{mech}$$

$$E^{[4]}_{E\ field\ 12} + E^{[4]}_{M\ field\ 12b} = -6E_{mech}$$

Field energy for n = 4

$$E'^{[4]}_{Mfield\ 12c} = \frac{1}{24c^4}\frac{d^4 I_1(t)}{dt^4}[\int da\ \hat{n}\cdot(\oint d\tilde{\vec{l_1}} R_1^3(t)\times\vec{B}_2'^{[0]})$$
$$+\ 4\pi I_2\oint\oint(d\vec{l_1}\cdot d\vec{l_2})R_{12}^3]$$

We notice that this magnetic energy term has a surface and volume contributions as follows:

$$E'^{[4]}_{Mfield\ 12c} = E'^{[4]}_{MfieldV\ 12c} + E'^{[4]}_{MfieldS\ 12c}$$

$$E^{[4]}_{MfieldV\ 12c} = \frac{\mu_0}{(4\pi)^2}E'^{[4]}_{MfieldV\ 12c} = \frac{\mu_0}{96\pi c^4}\frac{d^4 I_1(t)}{dt^4}I_2\oint\oint(d\vec{l_1}\cdot d\vec{l_2})R_{12}^3$$

It can easily seen that the change in volume energy is balance by the mechanical work done.

Field energy for n = 4

The surface terms of the field energy are:

$$
\begin{aligned}
E^{[4]}_{M\,fieldS\ 12c} &= \frac{\mu_0}{(4\pi)^2} E'^{[4]}_{M\,fieldS\ 12c} \\
&= \frac{\mu_0}{384\pi^2 c^4} \frac{d^4 I_1(t)}{dt^4} I_2 \int da\ \hat{n} \cdot \left(\oint d\tilde{\vec{l}}_1 R_1^3(t) \times \vec{B}'^{[0]}_2 \right)
\end{aligned}
$$

This term is not balanced by mechanical work and thus the only way to balance the derivative of this term in the energy equation is by a Poynting term which signifies the generation of radiation.

Field energy for n = 4

The total magnetic energy

$$E^{[4]}_{M field\ 12} = E^{[4]}_{M fieldV\ 12} + E^{[4]}_{M fieldS\ 12}$$

in which:

$$E^{[4]}_{M fieldV\ 12} = -4E_{mech} + \frac{\mu_0}{96\pi c^4} \frac{d^4 I_1(t)}{dt^4} I_2 \oint \oint (d\vec{l_1} \cdot d\vec{l_2}) R^3_{12}$$

and: $$E^{[4]}_{M fieldS\ 12} = E^{[4]}_{M fieldS\ 12c}$$

Field energy for n = 4

The total field energy will partitioned into a volume and surface terms as follows:

$$E^{[4]}_{field\ 12} = E^{[4]}_{fieldV\ 12} + E^{[4]}_{fieldS\ 12}$$

in which:

$$E^{[4]}_{fieldV\ 12} = -6E_{mech} + \frac{\mu_0}{96\pi c^4}\frac{d^4 I_1(t)}{dt^4} I_2 \oint \oint (d\vec{l_1} \cdot d\vec{l_2}) R^3_{12}$$

and:

$$E^{[4]}_{fieldS\ 12} = E^{[4]}_{MfieldS\ 12c}$$

It can easily seen that the change in volume energy is balance by the mechanical work done. However, the surface term remains unbalanced and cannot be balanced with a Poynting flux which indicate radiation.

Conclusions

- Newton's third law is not compatible with the principles of special relativity and the total force on a system of a current loop and a permanent magnet system is not zero.
- Still momentum and energy are conserved if one takes the field momentum and energy into account
- The energy needed for operation of relativistic engine comes at the expanse of the electromagnetic field energy.
- The total energy required is six times the mechanical energy obtained by the engine as energy must be invested also in driving the needed current for its operation through the loops.
- Two times comes at the expense of the electric field energy and four times at the expense of the magnetic field energy.

Photonic engine

- Finally we remark that although an energy of $6E_{mech}$ seems excessive and inefficient.

- It is highly efficient with respect to other types of engines which are purely electromagnetic.

- For example to reach a momentum p using a photon engine one needs an energy of $E_p = pc$.

- While for a relativistic engine an energy of $E_r = pv$ will suffice.

- The ratio ($\frac{E_p}{E_r} = \frac{c}{3v}$) is a huge number for non relativistic speeds.

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THANK YOU