Alternative Gravity in the Bottema 2015 and Carrignan 2013 Samples

J.G. O'Brien, T.L. Chiarelli, John T. Belanger

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Acknowledgements

- Thomas Chiarelli
- Philip Mannheim
- Stacey McGaugh
- Spasen Chaykov
- PLB

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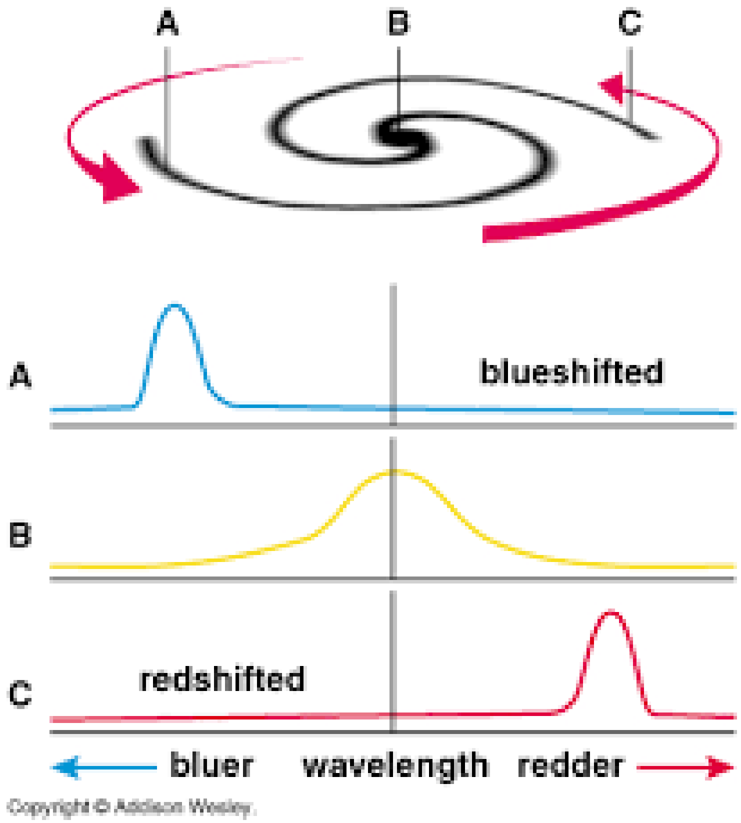
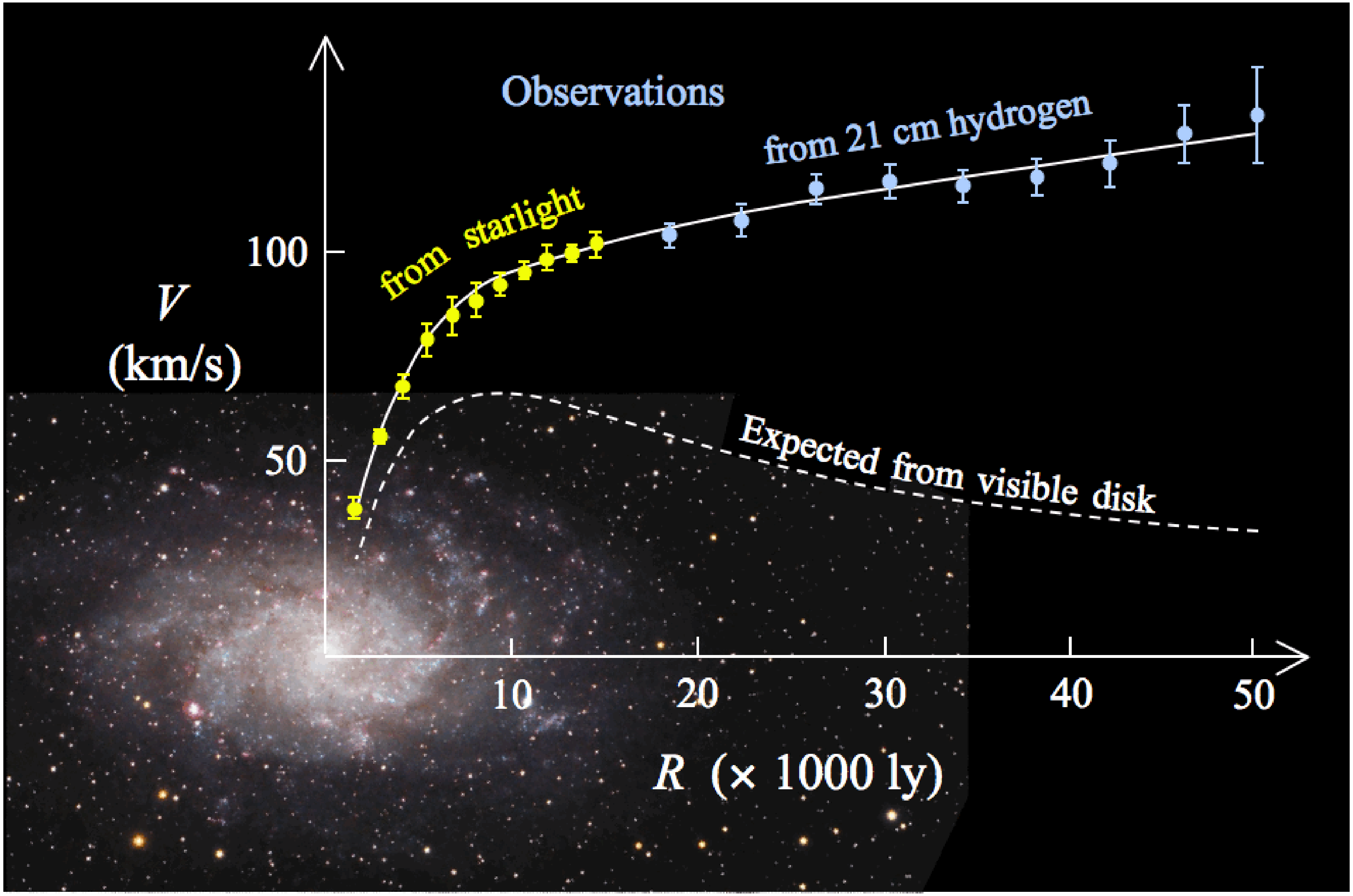
Overview

- This talk will highlight three features of the Bottema 2015 and Carrignan 2013 Surveys:

1. Rotation Curves for MOND, Conformal Gravity (CG) and MLS (to be explained).
2. Tully Fisher Analysis for MOND and CG.
3. Radial Acceleration Rule (RAR) for MOND, CG and MLS.

Review of Rotation Curves


A
B
C
A
blueshifted
B
C
redshifted
bluer
wavelength
redder
Copyright © Addison Wesley.



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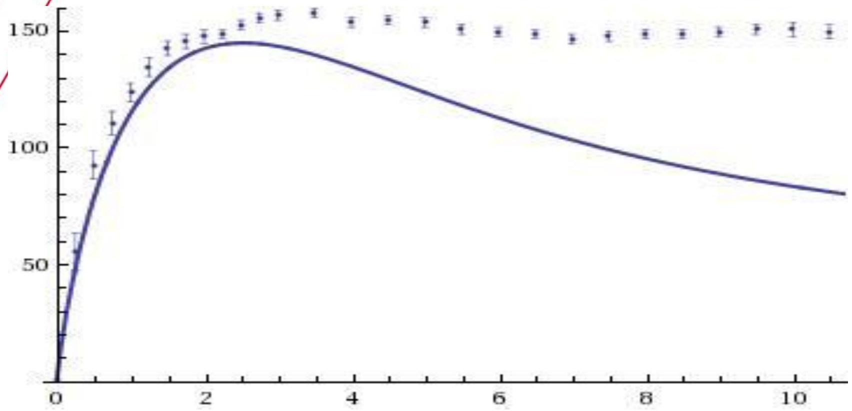
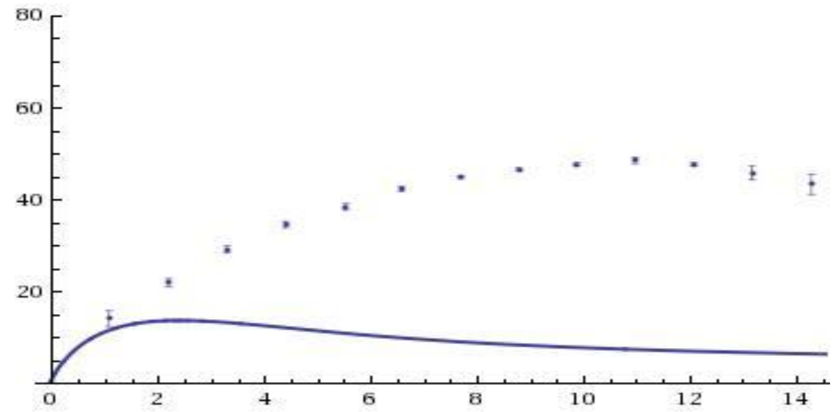
Some Real Examples: (Begeman1986)



NGC 3198

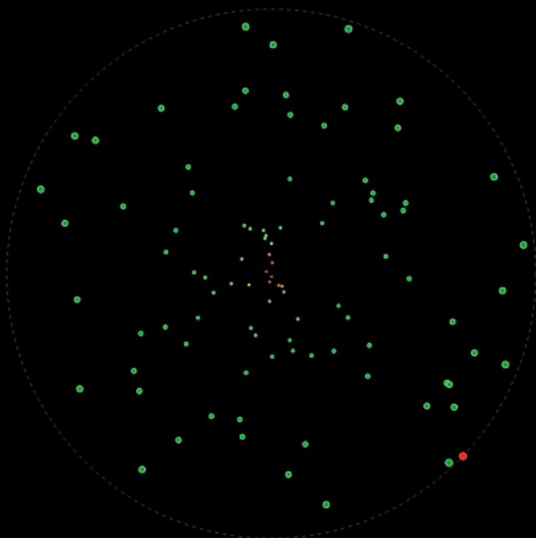


DDO 154

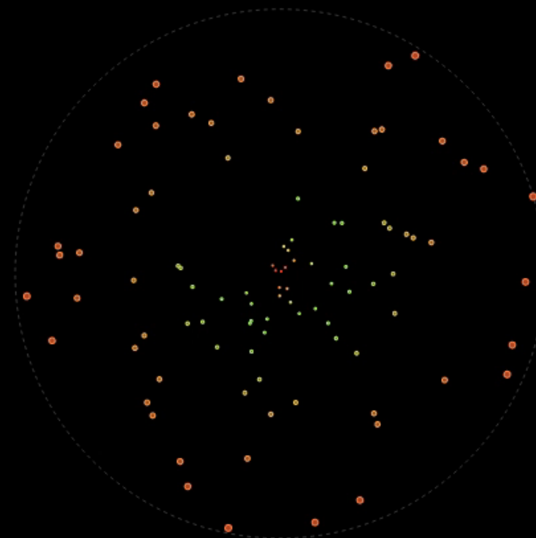
In each rotation curve, rotational velocity v is plotted against radial distance from the center of the galaxy.
In each of the above, we can see that the observation does not match the prediction.

Rotation Curve Simulation: MILKY-WAY Galaxy

Observational Data



General Relativity



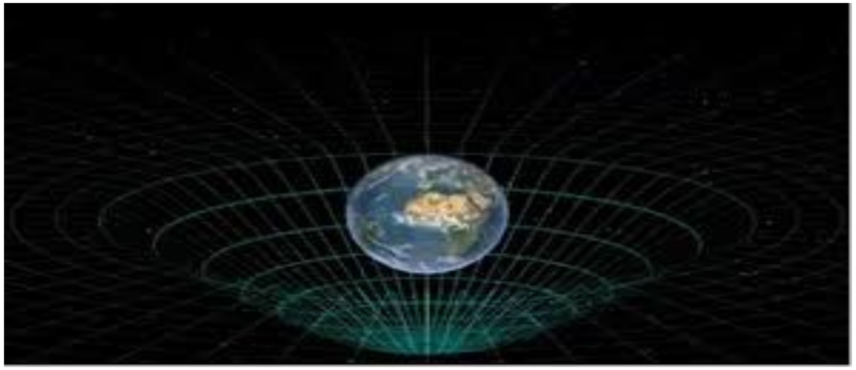
23 km/s 91 km/s 159 km/s

Reset animation spin speed

Distance (Mpc) (distance)	L_B (10^{10} $L_\odot$ W) (luminosity)	R_0 (kpc) (scale_length)	M_{HI} (10^{10} $M_\odot$ kg) (mass_hydrogen)	M_{disk} (10^{10} $M_\odot$ kg) (mass_disk)	$(M/L)_{stars}$ ($M_\odot/L_\odot$ kg/W) (mass_light_ratio)
14.1	3.241	4	1.06	3.6299	1.12

Point mass to galaxy disk

$$\beta^* = \left(1 - \frac{2GM}{r}\right)$$



$$V_\beta(R,z) = -\beta c^2 \int_0^\infty dR' \int_0^{2\pi} d\phi' \int_{-\infty}^{\infty} dz' \frac{R'\rho(R',z')}{(R^2 + R'^2 - 2RR'cos\phi' + (z-z')^2)^{1/2}}$$

$$\frac{v_{\rm lum}^2}{R} = g_\beta^{\rm lum} = \frac{N^*\beta^* c^2 R}{2R_0^3}\left[I_0\left(\frac{R}{2R_0}\right)K_0\left(\frac{R}{2R_0}\right) - I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\right]$$

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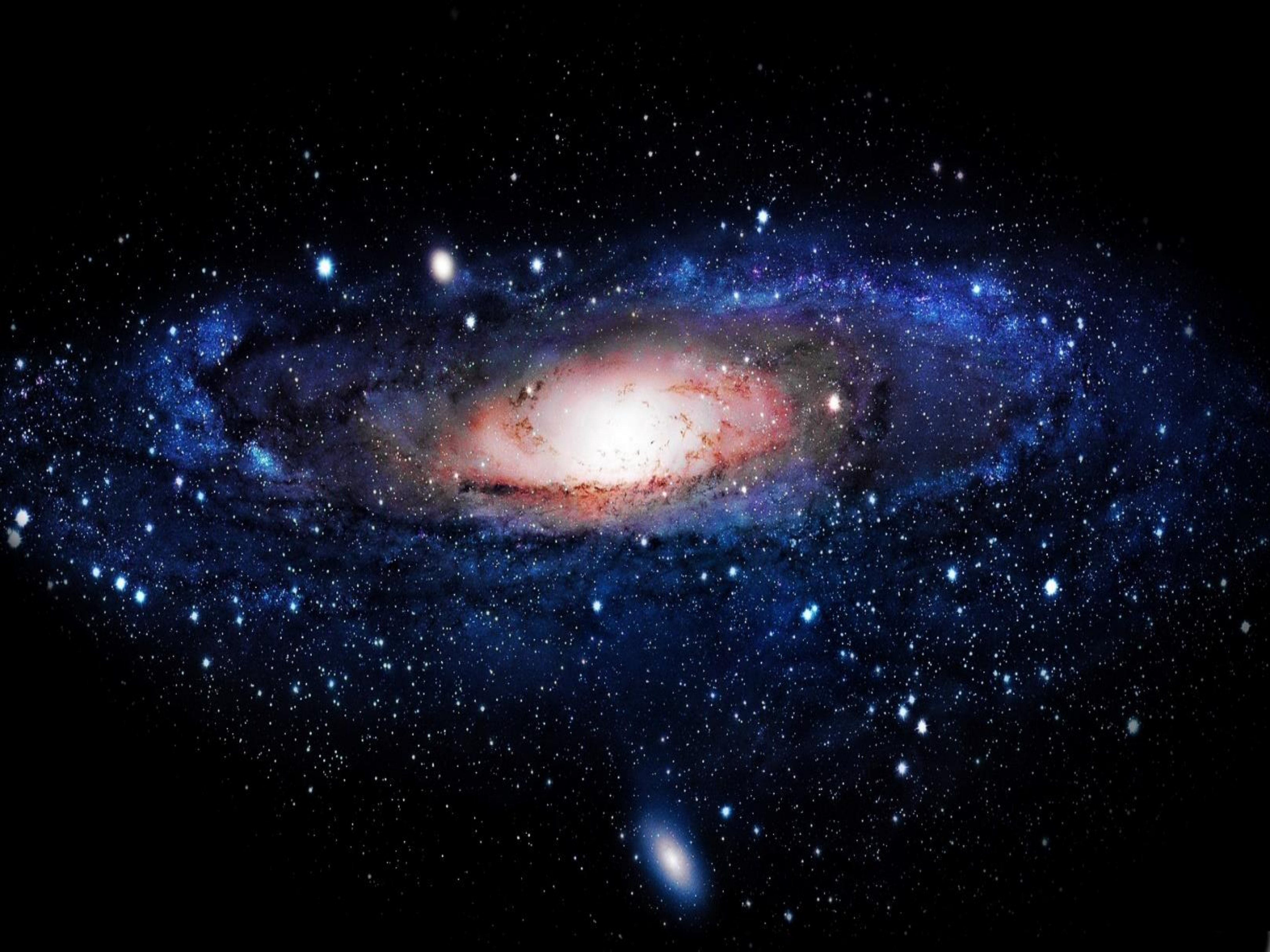
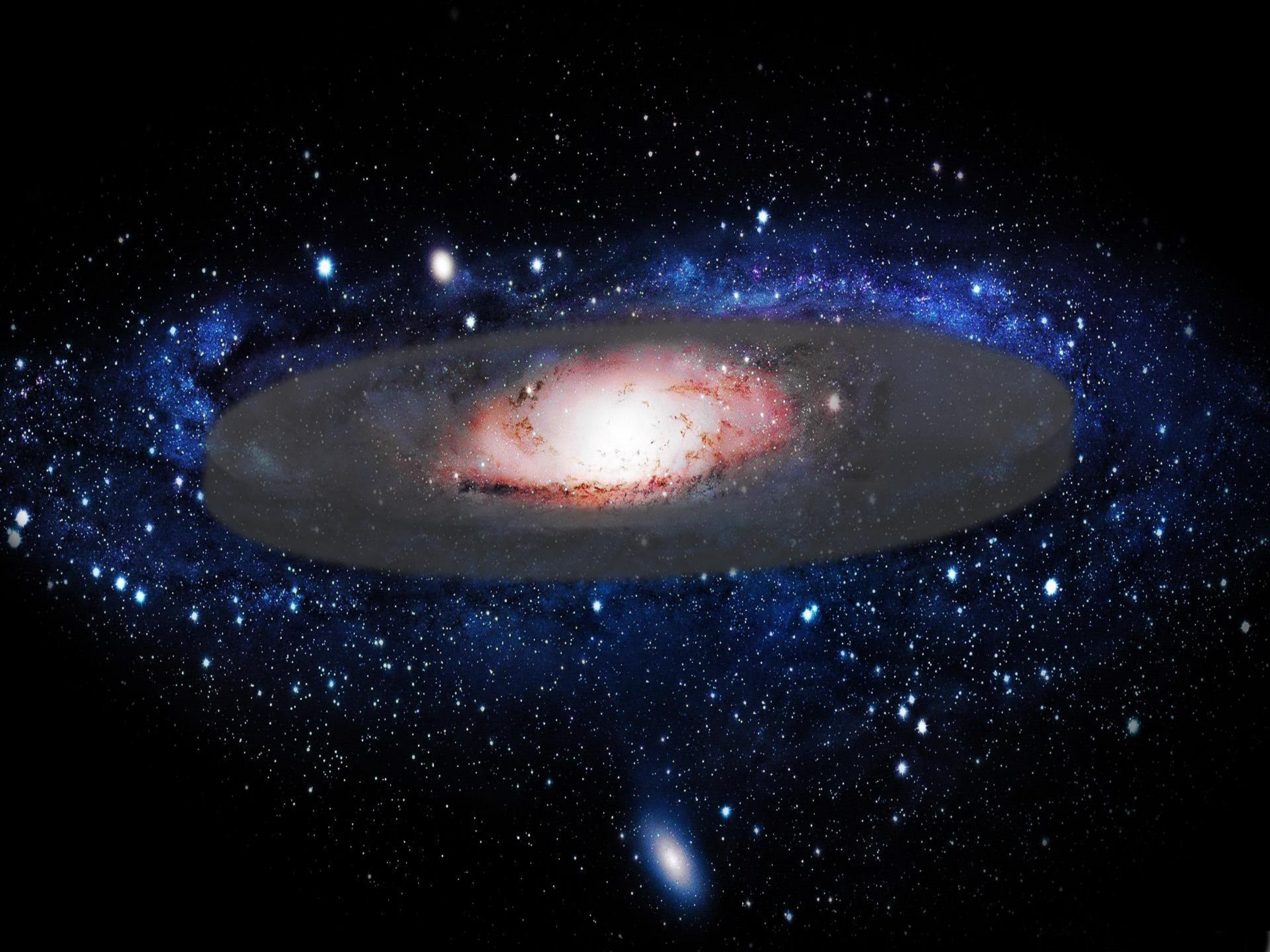
The Dark Matter Solution

- In order to solve this problem, if we observe the formula below, the only unknown is the mass. So what if we placed more mass in the galaxy at the right distance.

$$\frac{v_{\text{lum}}^2}{R} = g_\beta^{\text{lum}} = \frac{N^* \beta^* c^2 R}{2R_0^3} \left[I_0 \left(\frac{R}{2R_0} \right) K_0 \left(\frac{R}{2R_0} \right) - I_1 \left(\frac{R}{2R_0} \right) K_1 \left(\frac{R}{2R_0} \right) \right]$$

The only issue however is that if we do this, the mass would have to go inexactly the right place as to not overshoot the inner regions.





Dark Matter Formalism

- Thus, what if we add in a function which matches the flatness of the rotation curves at large distances, with two free parameters, a density and another falloff gradient,

$$\sigma(r) = \frac{\sigma_0}{(r^2 + r_0^2)} \qquad \frac{v_{\text{dark}}^2}{R} = g_\beta^{\text{dark}} = \frac{4\pi\beta^* c^2 \sigma_0}{R}\left[1 - \frac{r_0}{R}\arctan\left(\frac{R}{r_0}\right)\right]$$

The asymptotic limit (large radii) of the above yield the dark matter contribution to velocity as,

$$v_{\text{dark}}^2 \to 4\pi\beta^* c^2 \sigma_0 \qquad \sigma_0 = \frac{N^*}{10\pi R_0}$$

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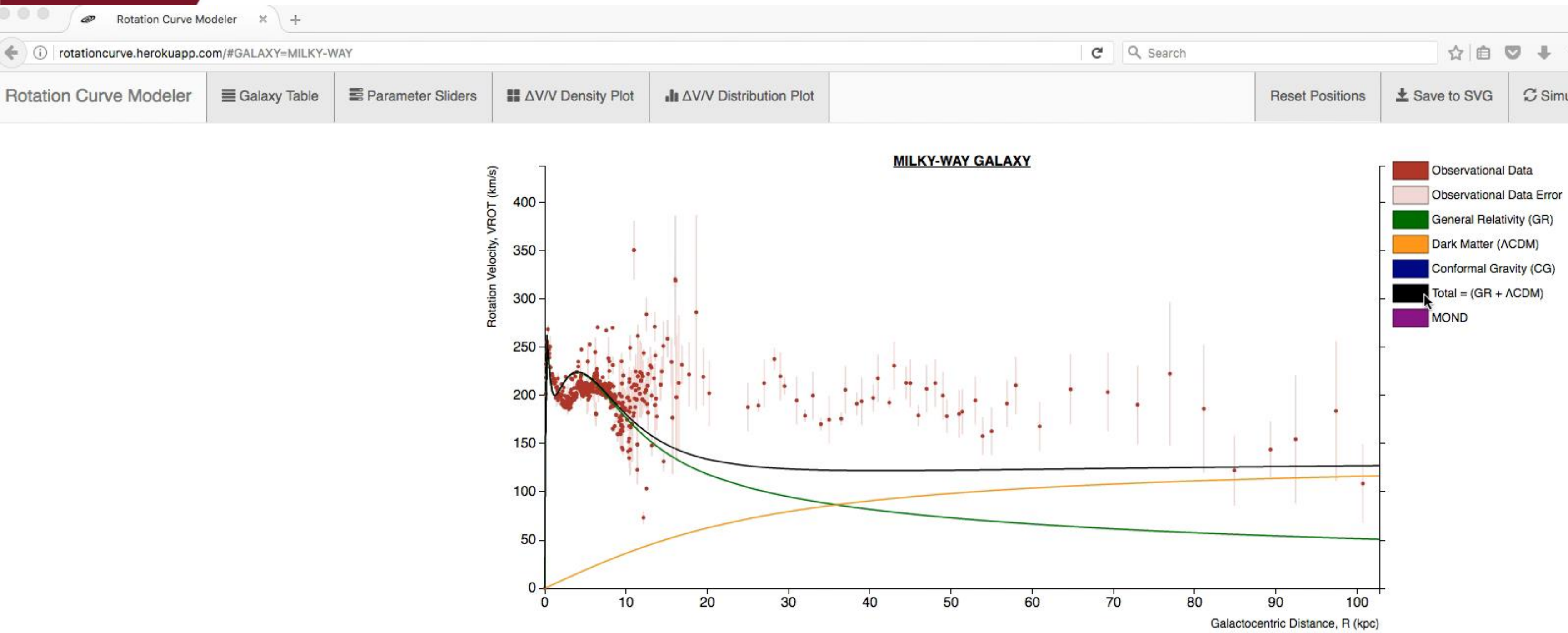

GR χ^2	CONFORMAL χ^2	TOTAL χ^2	MOND χ^2
6.70x10^5	1.36x10^6	6.55x10^5	5.90x10^6

Distance (Mpc) (distance)	L_B (10^{10} $L_\odot$ W) (luminosity)	R_0 (kpc) (scale_length)	M_{HI} (10^{10} $M_\odot$ kg) (mass_hydrogen)	M_{disk} (10^{10} $M_\odot$ kg) (mass_disk)	$(M/L)_{stars}$ ($M_\odot/L_\odot$ kg/W) (mass_light_ratio)
8.10x10^{-3}	1.62	2.1	1.18	5.527	3.41

$$\sigma(r) = \frac{\sigma_0}{(r^2 + r_0^2)} \qquad \frac{v_{\text{dark}}^2}{R} = g_\beta^{\text{dark}} = \frac{4\pi\beta^* c^2 \sigma_0}{R}\left[1 - \frac{r_0}{R}\arctan\left(\frac{R}{r_0}\right)\right] \quad v_{\text{dark}}^2 \rightarrow 4\pi\beta^* c^2 \sigma_0 \qquad \sigma_0 = \frac{N^*}{10\pi R_0}$$

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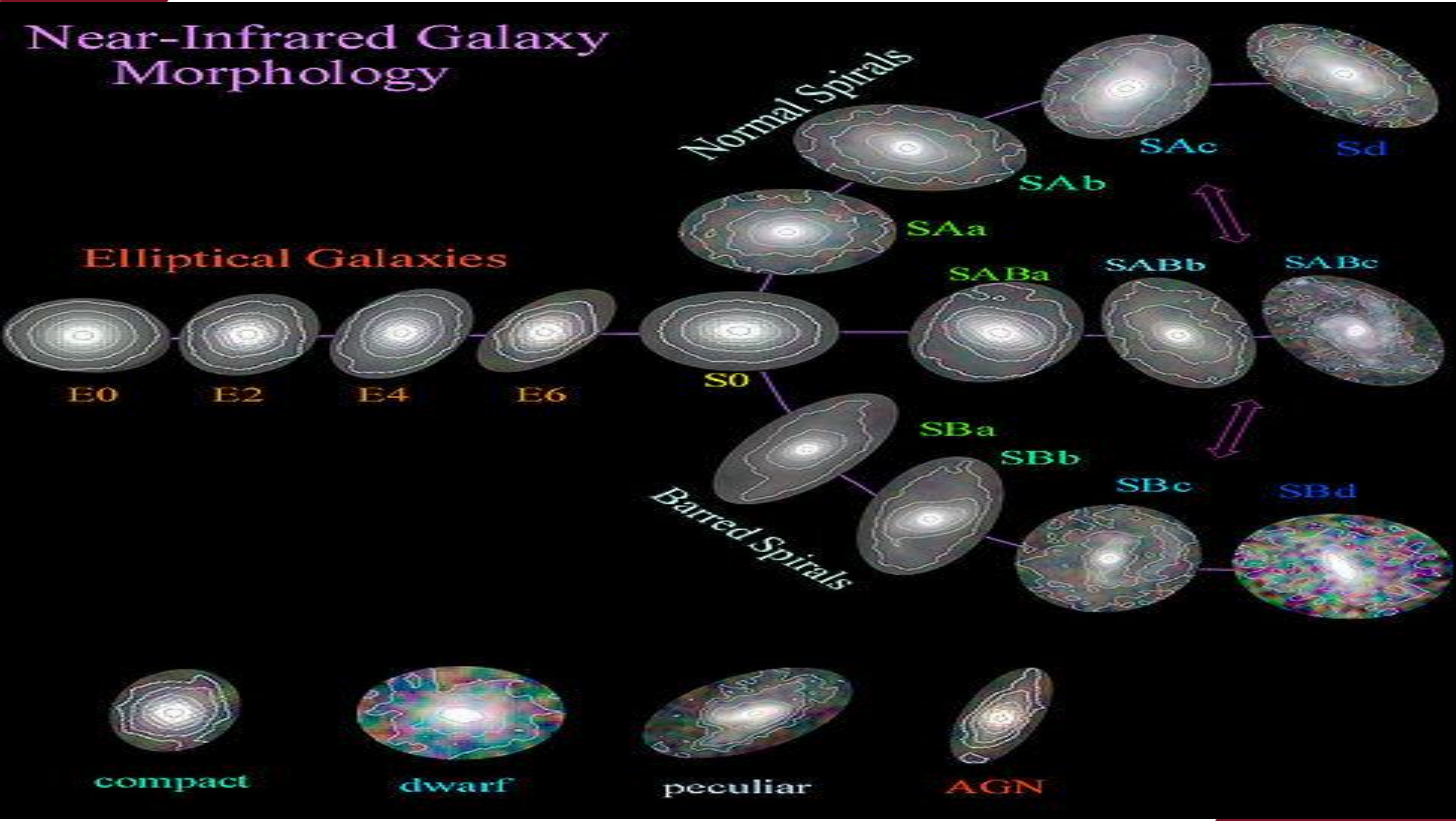
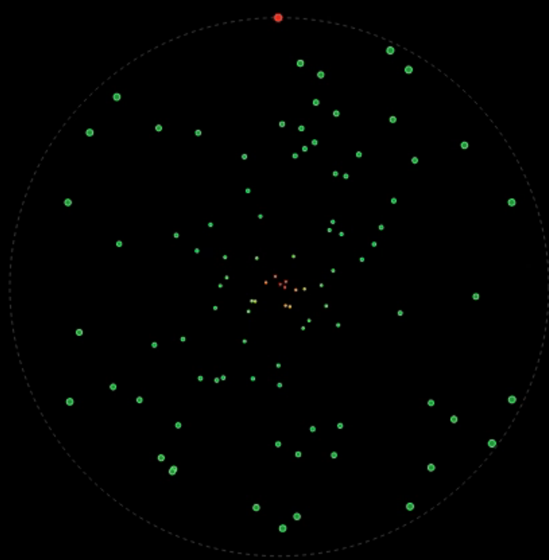
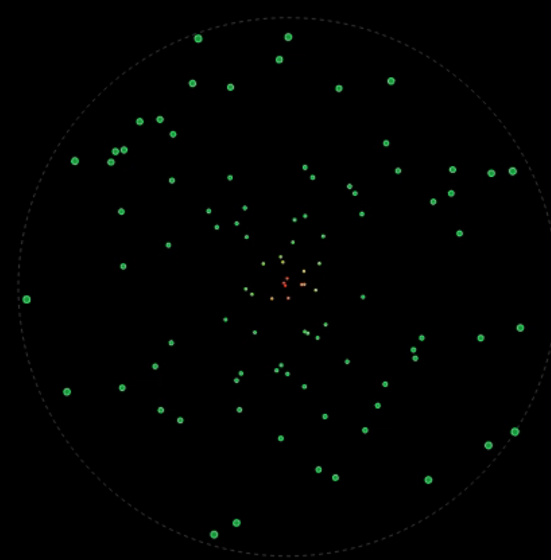

Near-Infrared Galaxy Morphology
Normal Spirals
SAa
SAb
SAc
Sd
Elliptical Galaxies
E0
E2
E4
E6
S0
SABa
SABb
SABc
SBa
SBb
SBc
SBd
Barred Spirals
compact
dwarf
peculiar
AGN

Rotation Curve Simulation: NGC-3198 Galaxy

Observational Data



Lambda Cold Dark Matter



23 km/s 91 km/s 159 km/s

Reset animation spin speed

Distance (Mpc) (distance)	L_B (10^{10} $L_\odot$ W) (luminosity)	R_0 (kpc) (scale_length)	M_{HI} (10^{10} $M_\odot$ kg) (mass_hydrogen)	M_{disk} (10^{10} $M_\odot$ kg) (mass_disk)	$(M/L)_{stars}$ ($M_\odot/L_\odot$ kg/W) (mass_light_ratio)
14.1	3.241	3.7642	1.06	2.9108	0.9

The Radial Acceleration Rule

Radial Acceleration Relation (RAR) of MLS

The Radial Acceleration Relation in Rotationally Supported Galaxies

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James M. Schombert
Department of Physics, University of Oregon, Eugene, OR 97403, USA
(Dated: September 21, 2016)

We report a correlation between the radial acceleration traced by rotation curves and that predicted by the observed distribution of baryons. The same relation is followed by 2693 points in 153 galaxies with very different morphologies, masses, sizes, and gas fractions. The correlation persists even when dark matter dominates. Consequently, the dark matter contribution is fully specified by that of the baryons. The observed scatter is small and largely dominated by observational uncertainties. This radial acceleration relation is tantamount to a natural law for rotating galaxies.

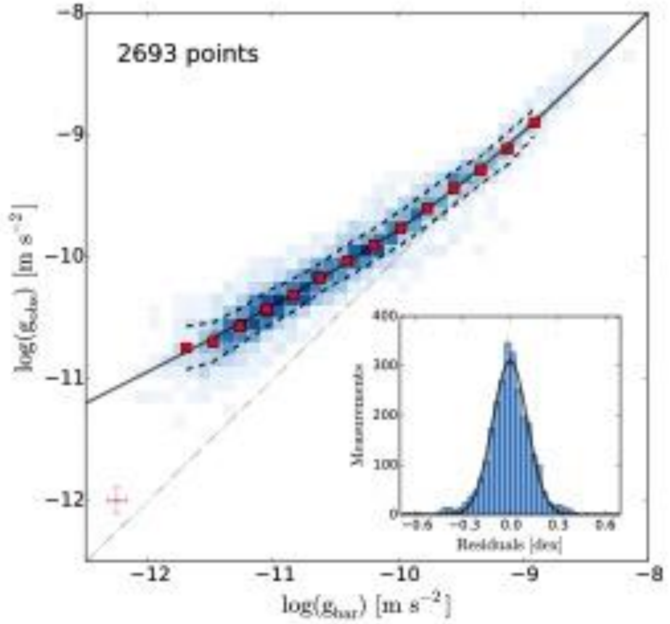
MLS - RAR

- Used the SPARC data set of 153 galaxies, 2693 data points in total.
- For each data point, showed:

$$g_{obs} = \frac{v_{obs}^2}{R_{obs}} \quad g_{bar} = \frac{v_{bar}^2}{R_{obs}}$$

- Plotted

$$g_{obs} \quad vs. \quad g_{bar}$$



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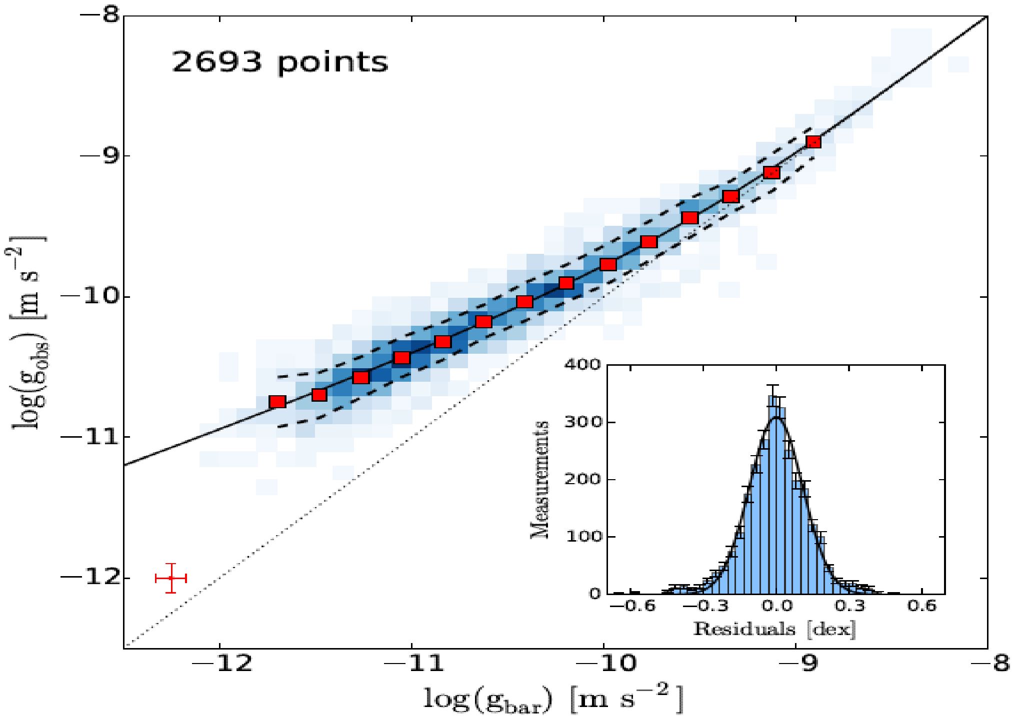
Centripetal Acceleration



Possible interpretations for the radial acceleration re-
ion fall into three broad categories.

1. It represents the end product of galaxy formation.
2. It represents new dark sector physics that leads to the observed coupling.
3. It is the result of new dynamical laws rather than dark matter.

$$g_{obs} = \frac{g_{bar}}{1 - e^{-\sqrt{\frac{g_{bar}}{g_+}}}}$$

$$g_{DM} = g_{obs} - g_{bar} = \frac{g_{bar}}{e^{\sqrt{\frac{g_{bar}}{g_+}}} - 1}$$

$$g_+ \approx a_0 = 1.2 * 10^{-}10cms^{-2}$$

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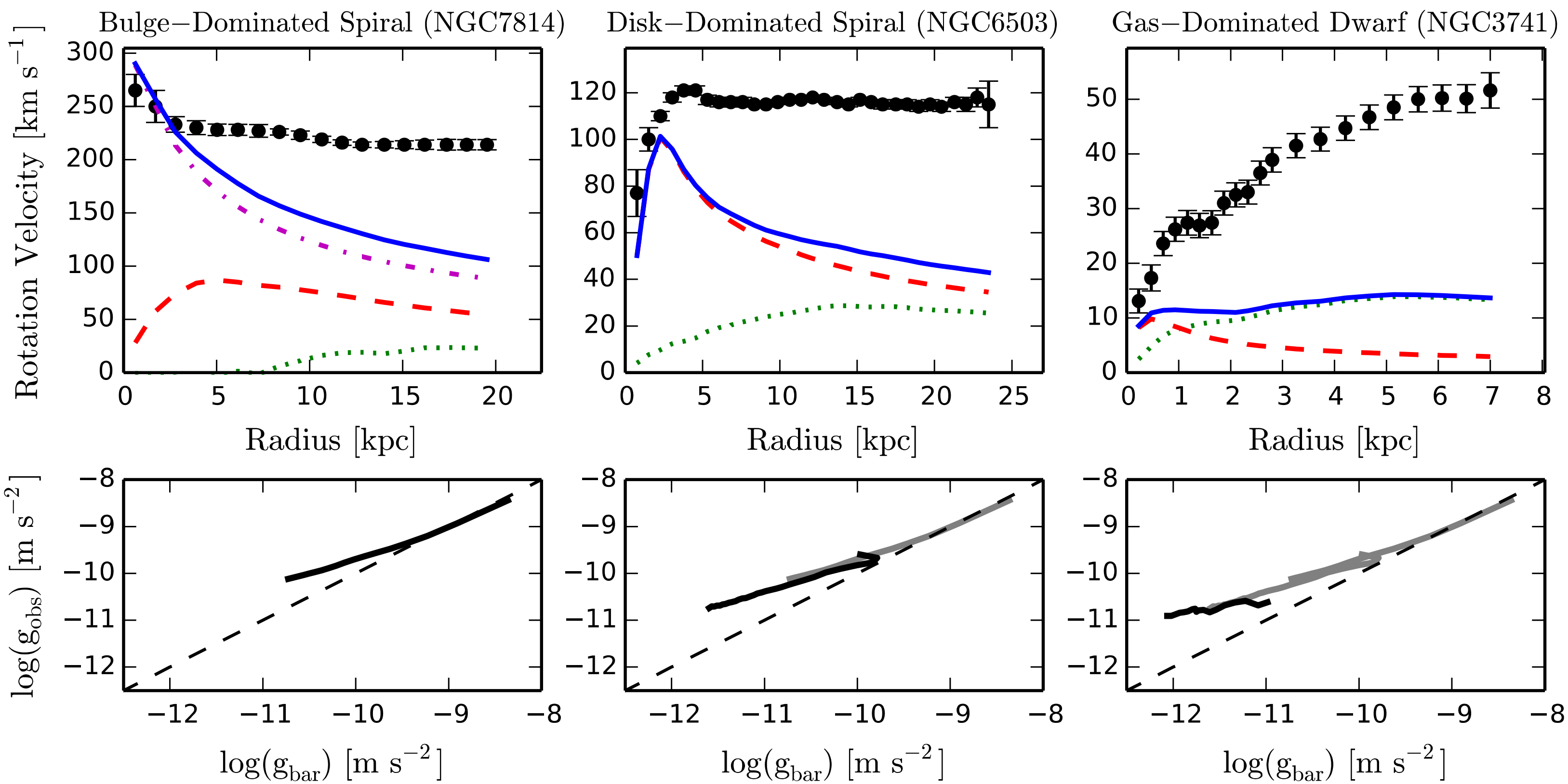
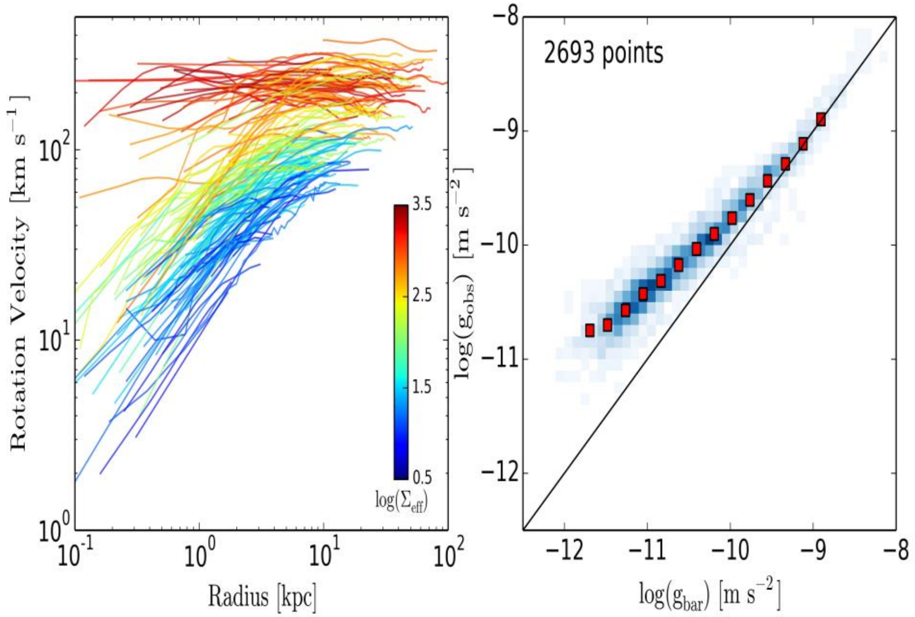
RAR



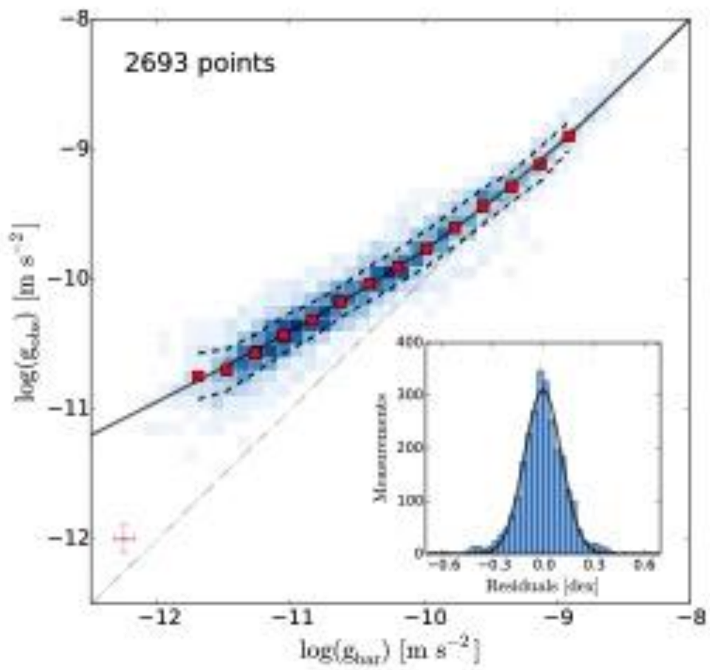
Bulge−Dominated Spiral (NGC7814)
Disk−Dominated Spiral (NGC6503)
Gas−Dominated Dwarf (NGC3741)
Rotation Velocity [km s−1]
Radius [kpc]
Radius [kpc]
Radius [kpc]
log(gobs) [m s−2]
log(gbar) [m s−2]
log(gbar) [m s−2]
log(gbar) [m s−2]

RAR



Rotation Velocity [km s^{-1}]
Radius [kpc]
log(Σ_{eff})
3.5
2.5
1.5
0.5
2693 points
log(g_{obs}) [m s^{-2}]
log(g_{bar}) [m s^{-2}]

MLS Solution Summary



$$g_{obs} = \frac{g_{bar}}{1 - e^{-\sqrt{\frac{g_{bar}}{g_+}}}}$$

$$g_{DM} = g_{obs} - g_{bar} = \frac{g_{bar}}{e^{\sqrt{\frac{g_{bar}}{g_+}}} - 1}$$

$$g_+ \approx a_0 = 1.2 * 10^{-}10cms^{-2}$$

From the above, we can back solve for the rotational velocity predicted by the baryons at a particular distance, which allows for the construction of rotation curves using the MLS fitting function. This will be used in fits belwo.

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Conformal Gravity

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Conformal Theory

The Conformal Theory was originally developed by Weyl, and later re-explored by Mannheim and Kazanas. It is a fourth order, scale invarient renormalizable gravitational theory:

$$
\begin{aligned}
I_W &= -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \\
&= -\alpha_g \int d^4x(-g)^{1/2} \left[R_{\lambda\mu\nu\kappa} R^{\lambda\mu\nu\kappa} - 2R_{\mu\kappa} R^{\mu\kappa} + \frac{1}{3}(R^\alpha{}_\alpha)^2 \right]
\end{aligned}
$$

where:

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left(g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu}\right) + \frac{1}{6} R^\alpha{}_\alpha \left(g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}\right)$$

$$(-g)^{-1/2} \delta I_W / \delta g_{\mu\nu} = -2\alpha_g W^{\mu\nu} = -T^{\mu\nu}/2$$

$$
\begin{aligned}
W^{\mu\nu} = {} & g^{\mu\nu} (R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/2 + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta} R^\nu{}_\beta + g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta}/2 \\
& -2g^{\mu\nu} (R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/3 + 2(R^\alpha{}_\alpha)^{;\mu;\nu}/3 + 2R^\alpha{}_\alpha R^{\mu\nu}/3 - g^{\mu\nu} (R^\alpha{}_\alpha)^2/6,
\end{aligned}
$$

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Conformal Theory

The Schwarzschild like solution in conformal theory can be solved via:

$$ds^2 = -b(\rho)dt^2 + a(\rho)d\rho^2 + \rho^2 d\Omega_2$$

$$\rho = p(r) \quad , \quad B(r) = \frac{r^2 b(r)}{p^2(r)} \quad , \quad A(r) = \frac{r^2 a(r) p'^2(r)}{p^2(r)}$$

$$-\frac{1}{p(r)} = \int \frac{dr}{r^2 [a(r)b(r)]^{1/2}}$$

$$ds^2 = \frac{p^2(r)}{r^2}\left[-B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_2\right] \quad .$$

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_2$$

Conformal Theory

$$\frac{W^{rr}}{B(r)} = \frac{B'B'''}{6} - \frac{B''^2}{12} - \frac{1}{3r}(BB''' - B'B'')$$
$$-\frac{1}{3r^2}(BB'' + B'^2) + \frac{2BB'}{3r^3} - \frac{B^2}{3r^4} + \frac{1}{3r^4} \quad ,$$

$$W^{00} = -\frac{B''''}{3} + \frac{B''^2}{12B} - \frac{B'''B'}{6B} - \frac{B'''}{r} - \frac{B''B'}{3rB}$$
$$+\frac{B''}{3r^2} + \frac{B'^2}{3r^2B} - \frac{2B'}{3r^3} - \frac{1}{3r^4B} + \frac{B}{3r^4}$$

$$\frac{3}{B}\left(W^0{}_0 - W^r{}_r\right) = B'''' + \frac{4B'''}{r} = \frac{1}{r}(rB)'''' = \nabla^4 B$$

$$B(r > R) = -g_{00} = \frac{1}{g_{rr}} = 1 - \frac{2\beta}{r} + \gamma r$$

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Conformal theory - Global

Since the conformal theory uses a fourth Poisson equation, we are not free to use only the local considerations as in Newtonian gravity.

$$\nabla^4\phi(r) = h(r) = f(r)c^2/2$$

$$\begin{aligned}
\phi(r) &= -\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') - \frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r'), \\
\frac{d\phi(r)}{dr} &= -\frac{1}{2}\int_0^r dr' r'^2 h(r') + \frac{1}{6r^2}\int_0^r dr' r'^4 h(r') - \frac{r}{3}\int_r^\infty dr' r' h(r'), \qquad (2.8)
\end{aligned}$$

We thus need to include a contribution from the cosmology, and in-homogeneities to the cosmology.

Cosmology term

We can implement a Robertson Walker metric in static coordinates via the following transformation

$$\rho = \frac{4r}{2(1+\gamma_0 r - kr^2)^{1/2} + 2 + \gamma_0 r}, \qquad \tau = \int dt R(t)$$

Brings the metric to the following form,

$$-(1+\gamma_0 r - kr^2)c^2 dt^2 + \frac{dr^2}{(1+\gamma_0 r - kr^2)} + r^2 d\Omega_2 =$$
$$\frac{1}{R^2(\tau)} \frac{[1-\rho^2(\gamma_0^2/16 + k/4)]^2}{[(1-\gamma_0\rho/4)^2 + k\rho^2/4]^2} \left[-c^2 d\tau^2 + \frac{R^2(\tau)}{[1-\rho^2(\gamma_0^2/16+k/4)]^2} \left(d\rho^2 + \rho^2 d\Omega_2\right) \right] \quad (2.10)$$

which we can see can be written as conformal to flat, as

$$ds^2 = e^{2\alpha(\tau,\rho)} \left[-c^2 d\tau^2 + \frac{R^2(\tau)}{[1+K\rho^2/4]^2} \left(d\rho^2 + \rho^2 d\Omega_2\right) \right],$$

Cosmology Term cont'd.

Since the transformed metric is conformally equivalent to a co-moving Robertson Walker Metric, with spatial curvature written below, then when written as a static coordinate system, the co-moving conformal cosmology behaves just like a static metric with universal linear and quadratic potentials.

With three space Curvature K= $-\gamma_0^2/4 - k.$

$$\frac{v_{\mathrm{TOT}}^2}{R} = \frac{v_{\mathrm{LOC}}^2}{R} + \frac{\gamma_0 c^2}{2}.$$

In Mannheim's original fits, the k (quadratic term) was left out, so that:

$$-(1+\gamma_0 r - kr^2)c^2dt^2 \Longrightarrow -(1+\gamma_0 r)c^2dt^2$$

So in a topologically open RW cosmology, we introduce the universal linear potential, hence

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Conformal Gravity

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$$\begin{aligned} I_W &= -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \\ &= -\alpha_g \int d^4x(-g)^{1/2} \left[R_{\lambda\mu\nu\kappa} R^{\lambda\mu\nu\kappa} - 2R_{\mu\kappa}R^{\mu\kappa} + \frac{1}{3}(R^\alpha{}_\alpha)^2 \right] \end{aligned}$$

where:

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}\left(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right) + \frac{1}{6}R^\alpha{}_\alpha\left(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right)$$

$$(-g)^{-1/2}\delta I_W/\delta g_{\mu\nu} = -2\alpha_g W^{\mu\nu} = -T^{\mu\nu}/2$$

$$\begin{aligned} W^{\mu\nu} = g^{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/2 + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta}R^\nu{}_\beta + g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}/2 \\ -2g^{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/3 + 2(R^\alpha{}_\alpha)^{;\mu;\nu}/3 + 2R^\alpha{}_\alpha R^{\mu\nu}/3 - g^{\mu\nu}(R^\alpha{}_\alpha)^2/6, \end{aligned}$$

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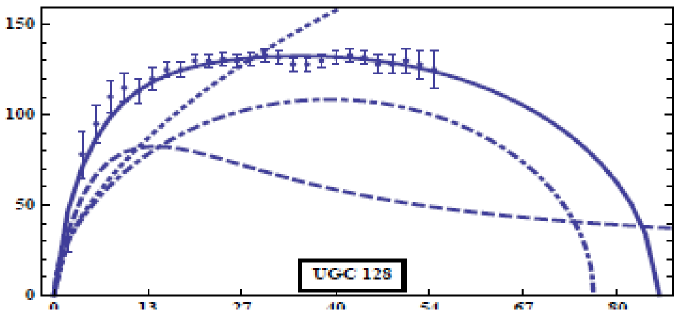
Rotation Curves in CG

- Summary: The total conformal gravity curve (local and global combined:

$$v_{cg}(R) = \sqrt{v_{gr}^2 + \frac{N^*\gamma^* c^2 R}{2R_0} F_{\gamma^*}(R) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2},$$

$$F_{\gamma^*}(R) = I_1\left(\frac{R}{2R_0}\right) K_1\left(\frac{R}{2R_0}\right).$$

$$\gamma^* = 5.42\mathrm{e}^{-41}\mathrm{cm}^{-1},$$
$$\gamma_0 = 3.06\mathrm{e}^{-30}\mathrm{cm}^{-1},$$
$$\kappa = 9.54\mathrm{e}^{-54}\ \mathrm{cm}^{-2}.$$



The Samples

- Bottema et. Al. 2015
- Carrignan et. Al. 2013

Bottema 2015

- Survey Consisting of 12 “high resolution”, well studied rotation curves.
- Survey spans a diverse mix of dwarf and large spiral galaxies.
- All galaxies have “well known” distances.
- Some galaxies in the sample are bulge galaxies.
- “Problematic” for alternative gravity (MOND).

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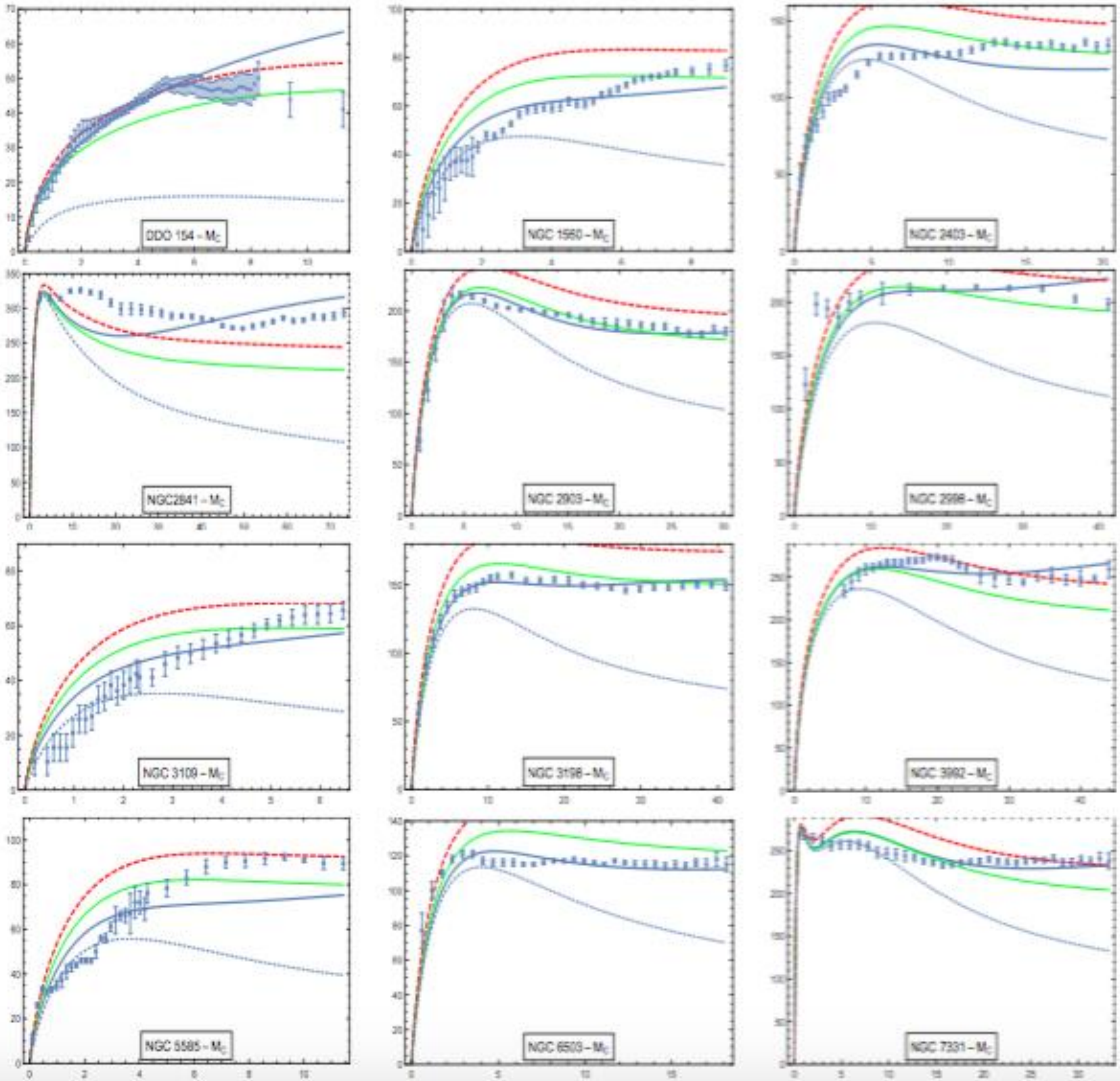
Table 1. Properties of the Bottema 2015 Galaxy Sample

Galaxy	Dist. (Mpc)	Lum. ($10^9 L\odot$)	R_0 (kpc)	M_{gas} ($10^9 M_\odot$)	$(M_d)_C$ ($10^9 M_\odot$)	$(M/L)_C$ ($M_\odot L_\odot^{-1}$)	$(v^2/c^2 R_l)_{last}$ ($10^{-30} cm^{-1}$)	$(M_d)_M$ ($10^9 M_\odot$)	$((M/L))M$ ($M_\odot L_\odot^{-1}$)
DDO 154	4.3	0.04	0.8	0.45	0.03	0.72	5.37	0.02	0.36
NGC 1560	3.2	0.70	1.4	0.95	1.78	2.55	2.35	0.64	0.91
NGC 2403	3.3	10.12	2.1	4.67	19.29	1.91	3.18	9.47	0.94
NGC 2841	16.2	101.61	4.0	37.69	152.76	1.50	4.25	218.39	2.15
NGC 2903	7.9	38.68	2.6	4.87	66.56	1.72	3.86	44.60	1.15
NGC 2998	59.6	75.52	4.8	23.91	91.13	1.21	3.42	68.27	0.90
NGC 3109	1.3	0.28	1.2	0.43	0.87	3.09	2.40	0.36	1.29
NGC 3198	12.8	24.53	3.6	12.86	36.64	1.49	1.94	18.33	0.75
NGC 3992	20.6	72.53	4.6	5.52	156.22	2.15	5.50	134.80	1.86
NGC 5585	7.4	2.48	1.6	1.22	2.90	1.17	2.52	1.00	0.40
NGC 6503	4.8	4.83	1.4	1.42	14.35	2.97	2.62	6.76	1.40
NGC 7331	13.4	49.80	3.0	141.62	1198.45	2.41	6.13	97.58	1.96

Columns: Galaxy Name, Distance (NED Average with no TF), Scale Length, Gas Mass (including helium), CG predicted Mass, M/L ration using CG mass, scaled centripetal acceleration of last data point, MOND predicted mass, M/L using MOND mass.

Plots

- Next slide shows the Bottema galaxies with the Conformal gravity predicted mass.
- Blue Curve=Conformal Gravity
- Red Curve=MOND
- Green Curve=MLS
- Dashed=Newtonian no dark matter

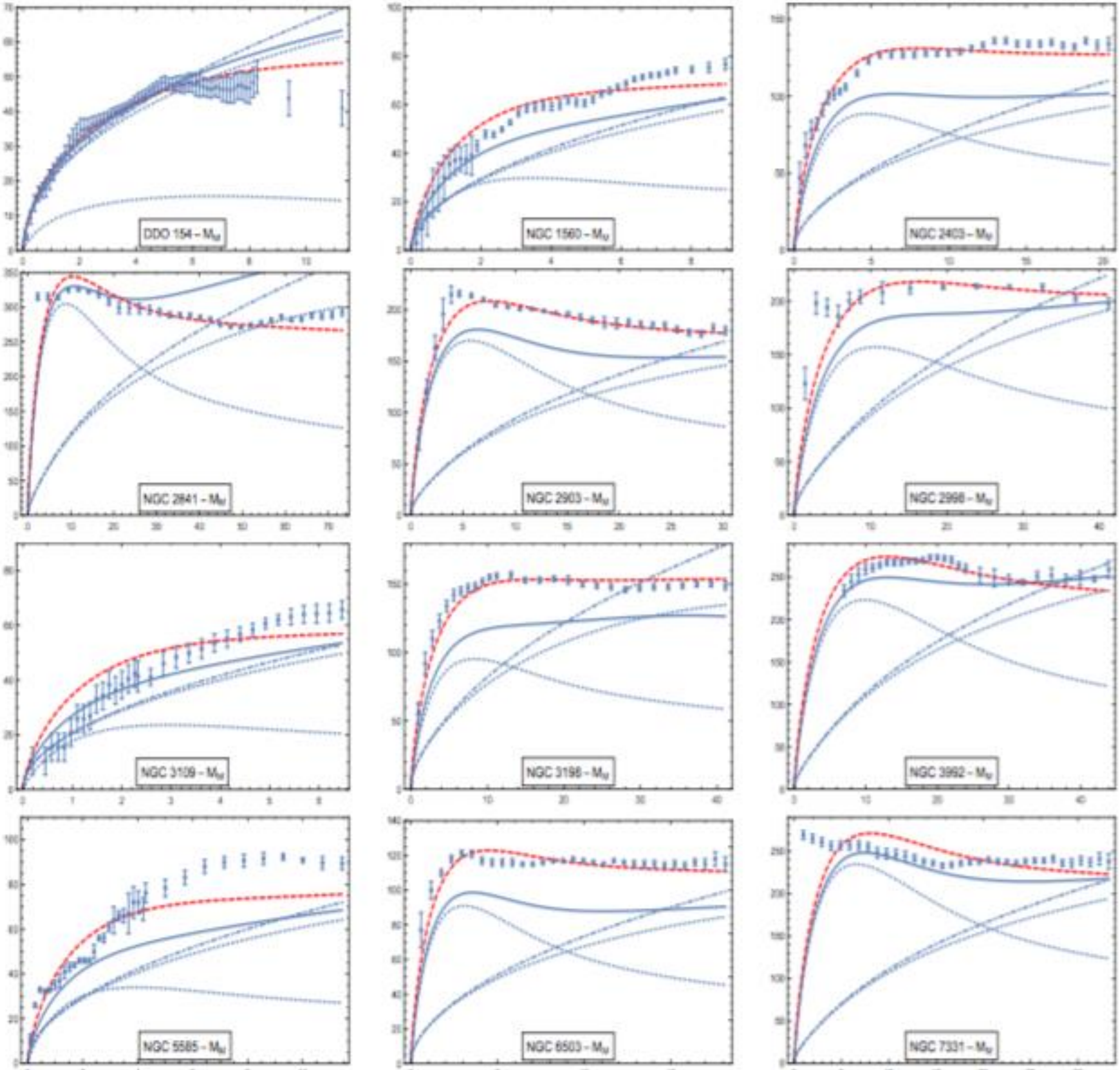


(M/L) $(M_{\odot} L_{\odot}^{-1})$
0.72
2.55
1.91
1.50
1.72
1.21
3.09
1.49
2.15
1.17
2.97
2.41

Blue=CG
Red=MOND
Green=MLS
Dashed=Ne
wtonian

Plots

- Next slide shows the Bottema galaxies with the MONDpredicted mass.
- Blue Curve=Conformal Gravity
- Red Curve=MOND
- Green Curve=MLS
- Dashed=Newtonian no dark matter



$(M/L)_M$ $(M_\odot L_\odot^{-1})$
0.23
1.17
1.32
2.37
1.65
0.90
0.74
2.18
5.31
4.82
0.38
1.57
2.62
4.77
0.48

Blue=CG
Red=MOND
Green=MLS
Dashed=Ne
wtonian

Carrignan 2013

- Survey Consisting of 15 “high resolution”, well studied rotation curves (some overlap with Bottema).
- Survey spans a diverse mix of dwarf and large spiral galaxies.
- Used different distance methods than Bottema.
- Some galaxies in the sample are bulge galaxies.
- “Problematic” for alternative gravity (MOND).

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Table 2. Properties of the Carignan 2013 Galaxy Sample

Galaxy	Dist. (Mpc)	Lum. ($10^9 L_\odot$)	R_0 (kpc)	M_{gas} ($10^9 M_\odot$)	$(M_d)_C$ ($10^{10} M_\odot$)	$(M/L)_C$ ($M_\odot L_\odot^{-1}$)	$(v^2/c^2 R_l)_{last}$ ($10^{-30} cm^{-1}$)	$(M_d)_M$ ($10^{10} M_\odot$)	$(M/L)_M$ ($M_\odot L_\odot^{-1}$)
DDO 154	4.3	0.08	0.8	0.44	0.03	0.38	1.23	0.02	0.23
IC 2574	3.3	0.80	3.1	1.42	2.29	2.87	2.15	0.94	1.17
NGC 55	1.9	0.81	1.9	1.81	2.93	3.63	2.19	1.06	1.32
NGC 247	3.3	2.17	3.8	1.85	12.28	5.66	3.27	5.14	2.37
NGC 300	1.9	1.74	2.0	1.06	7.00	4.01	1.37	2.88	1.65
NGC 925	7.8	5.12	3.5	4.67	11.44	2.23	4.21	4.62	0.90
NGC 2366	3.2	0.38	1.5	0.97	0.83	2.17	2.10	0.28	0.74
NGC 2403	3.4	4.77	2.1	4.02	21.56	4.52	3.58	10.37	2.18
NGC 2841	16.2	37.93	4.0	15.98	204.58	5.39	5.05	201.41	5.31
NGC 3031	3.7	14.40	2.6	5.25	85.45	5.93	9.31	69.38	4.82
NGC 3109	1.3	0.19	1.2	0.66	0.14	0.71	2.52	0.07	0.38
NGC 3198	12.8	11.53	3.6	12.31	36.63	3.18	2.29	18.13	1.57
NGC 3621	6.7	5.37	2.6	10.26	28.05	5.22	3.50	14.08	2.62
NGC 7331	13.4	20.38	3.0	10.67	120.47	5.91	10.14	97.19	4.77
NGC 7793	3.9	6.51	1.3	1.24	5.69	0.87	4.02	3.10	0.48

Columns: Galaxy Name, Distance (NED Average with no TF), Scale Length, Gas Mass (including helium), CG predicted Mass, M/L ration using CG mass, scaled centripetal acceleration of last data point, MOND predicted mass, M/L using MOND mass.

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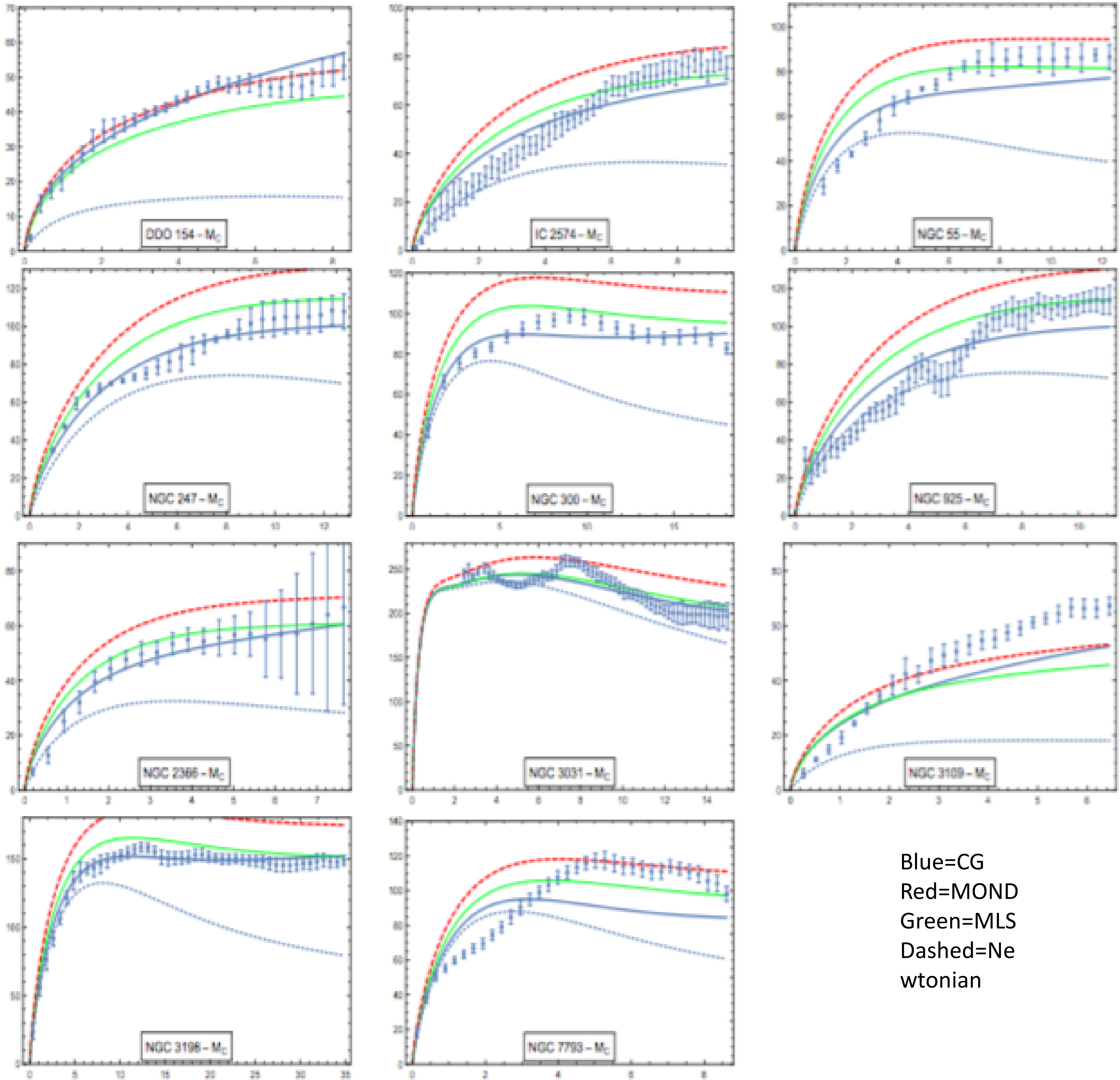
Plots

- Next slide shows the Carrignan galaxies with the Conformal gravity predicted mass.
- Blue Curve=Conformal Gravity
- Red Curve=MOND
- Green Curve=MLS
- Dashed=Newtonian no dark matter



Blue=CG
Red=MOND
Green=MLS
Dashed=Ne
wtonian

Plots

- Next slide shows the Carrignan Sample with the MOND predicted mass.
- Blue Curve=Conformal Gravity
- Red Curve=MOND
- Green Curve=MLS
- Dashed=Newtonian no dark matter

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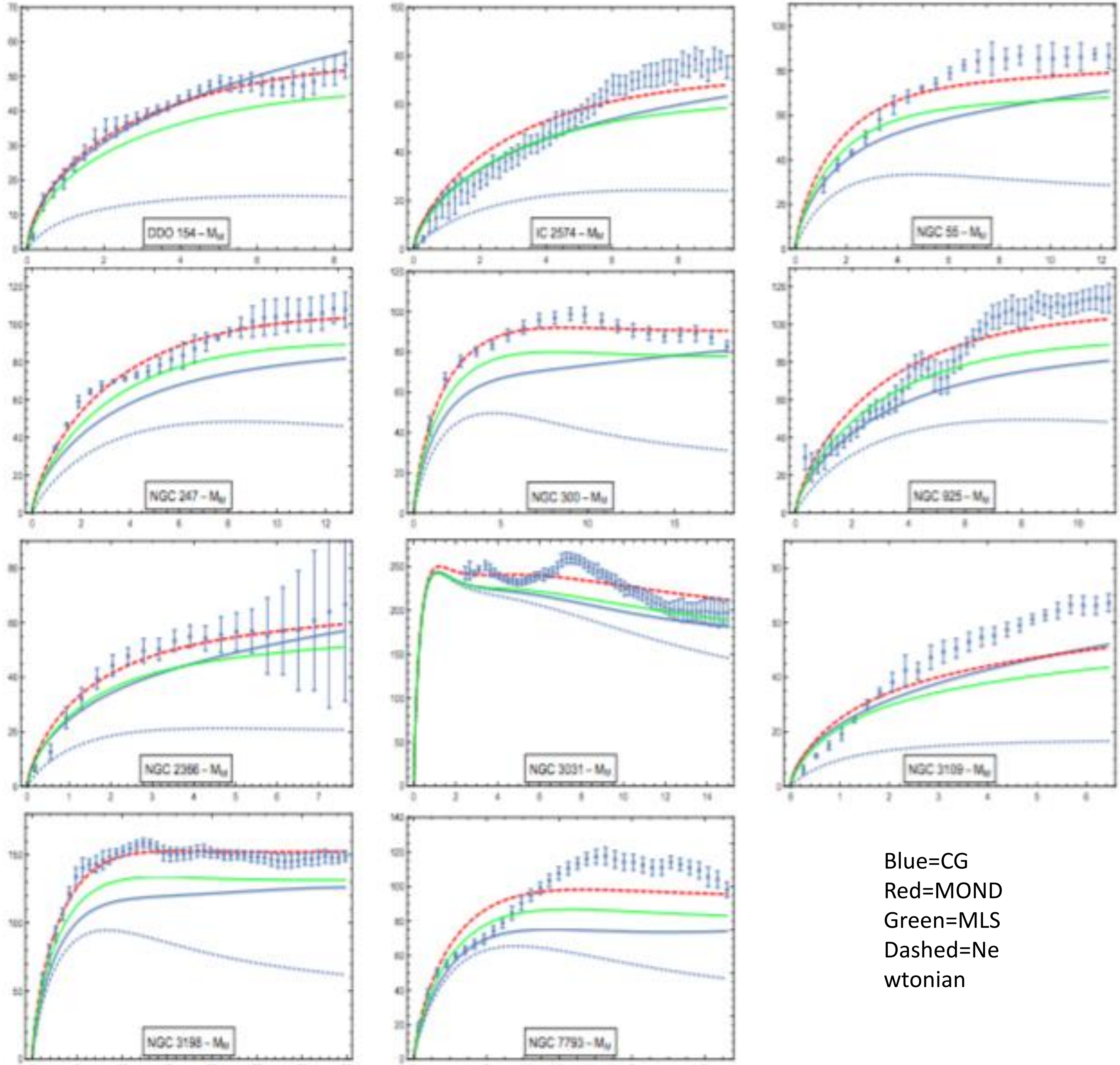

Blue=CG
Red=MOND
Green=MLS
Dashed=Ne
wtonian

Tully Fisher Relation

- An Empirical Observation that shows that for "flat or flattening" rotation curves,

$$v^4 \approx M$$

  in the "outer regions".
- There is currently no formalism for this, simply an observation
- Similar to Kepler's 3rd law before Newton.

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Notes about Tully Fisher

- Currently no "prediction" in standard theory.
- Has been more recently adopted to being called the "Baryonic Tully Fisher" relation, which includes gas and stars into the relation (helps with dwarf galaxies to fit the TF)
- MOND and standard theory typically use TF to bound mass and or constrain distance**
- Conformal gravity has a testable and falsifiable deviation from pure TF.

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TF in CG

$$v_{cg}(R) = \sqrt{v_{gr}^2 + \frac{N^*\gamma^* c^2 R}{2R_0} F_{\gamma^*}(R) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2},$$

Take the Asymptotic Limit, and neglect the k term.

$$v^2 = \beta^* c^2 N^*/R + (\gamma^* N^* + \gamma_0) c^2 R/2$$

Such that

$$v^4 = B(M/M_\odot)(1 + N^*/D)$$

where

$$B = 2c^2 M_\odot G \gamma_0 = 0.0074 km^4 s^{-4}$$
$$D = \gamma_0/\gamma^* = 5.65 \times 10^{10}$$

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TF

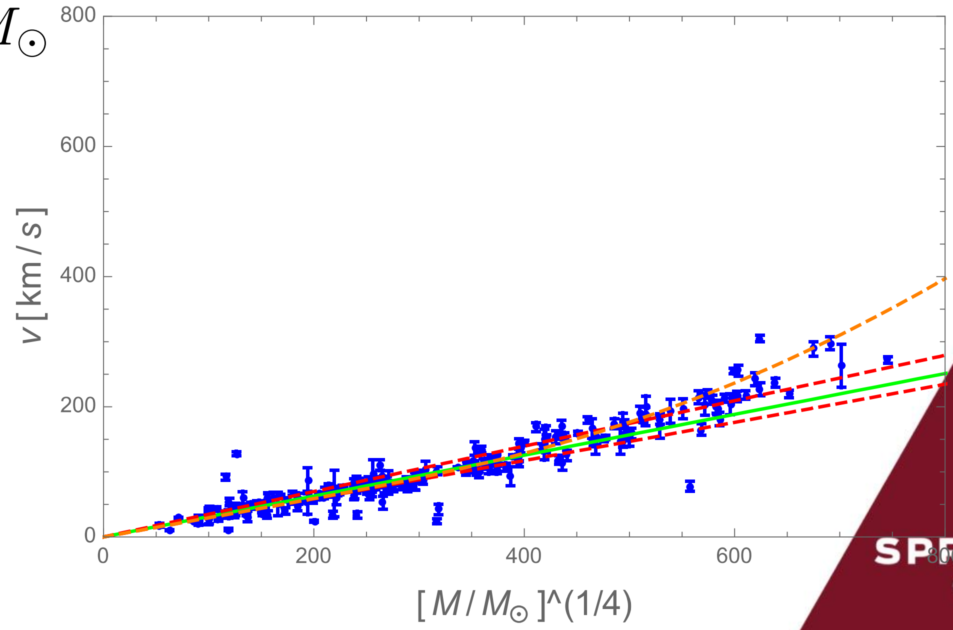
$$v^4 = B(M/M_\odot)(1 + N^*/D)$$

$$B = 2c^2 M_\odot G\gamma_0 = 0.0074 km^4 s^{-4}$$

$$D = \gamma_0/\gamma^* = 5.65 \times 10^{10}$$

Since most galaxies, have $N^* < 5.65 \times 10^{10}$

$$BM/M_\odot < v^4 < 2BM/M_\odot$$



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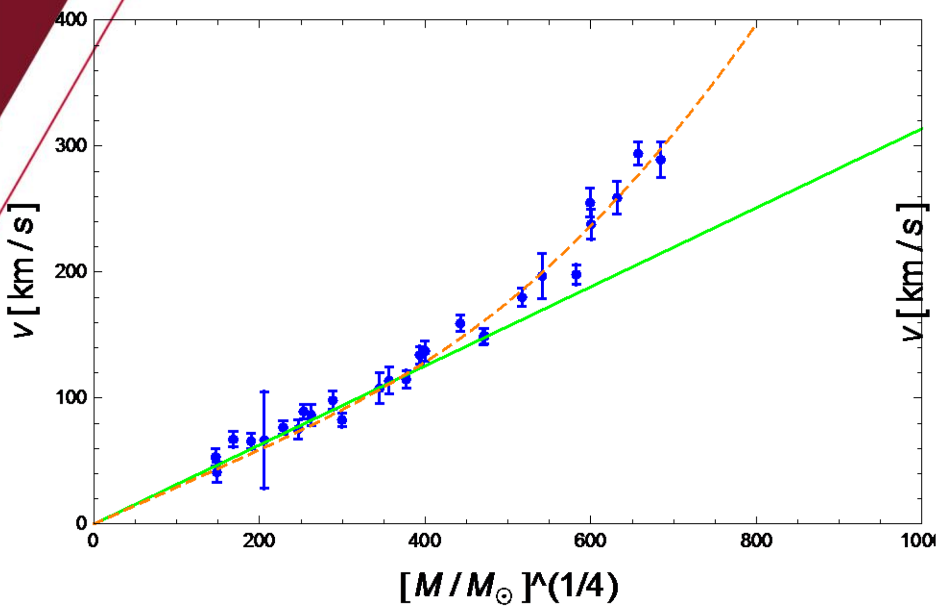
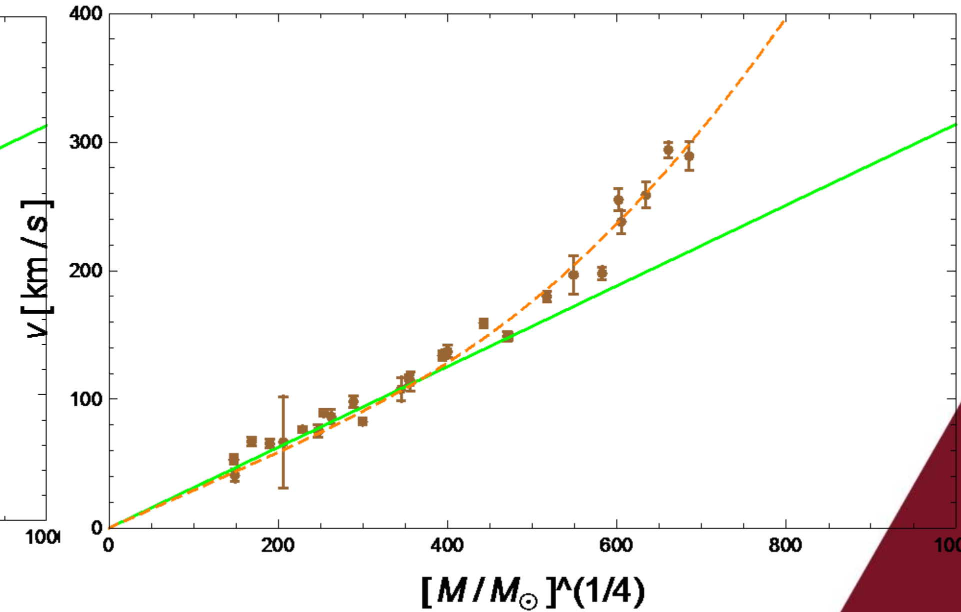
Analysis of the two samples





Above: TF relation for the last data point in each galaxy using the CG predicted mass.
Green=Pure TF
Orange=CG deviation from TF

Above: TF relation for the last data point in each galaxy using the MOND predicted mass.
Green=Pure TF
Orange=CG deviation from TF

Summary of TF

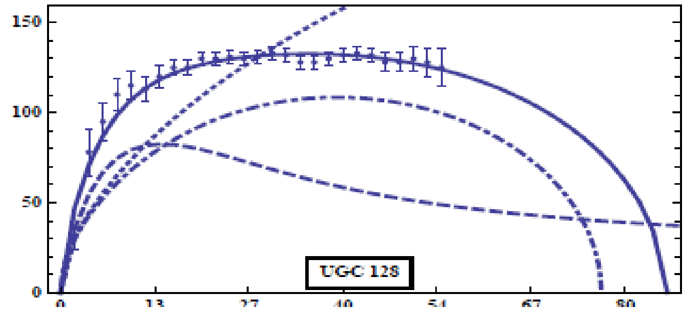
- Using latest distances that do not reference TF to find the distance, only few galaxies show deviation from pure TF.
- Both predicted masses show deviation from TF in large galaxies that fit with the CG deviation.
- Larger galaxies using cepheids may lead to more observable deviations.

Importance of the Deviation

What we are proposing here is two-fold for TF:

1. CG can explain the relationship and provide bounds in a straightforward manner.
2. If we accept this premise, we can perhaps use CG as a method for removing an age old problem in Rotation Curve Physics, namely distance estimates.

$$v_{cg}(R) = \sqrt{v_{gr}^2 + \frac{N^*\gamma^* c^2 R}{2R_0} F_{\gamma^*}(R) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2},$$



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Individually Referenced Moduli and Distances for NGC 2841 (as published)

(m-M)	err(m-M)	D(Mpc)	Method	REFCODE	Notes	SN Name	Redshift	H_0 (km/s/Mpc)	Adopted LMC modulus
30.58	0.06	13.100	Cepheids	2001ApJ...559..243M	18, VI				
30.74	0.23	14.100	Cepheids	2001ApJ...559..243M	18, VI, Zcor				
30.75	0.06	14.100	Cepheids	2006ApJS..165..108S	LMC ZP				18.54
30.75		14.100	Cepheids	2008A&ARv..15..289T					18.54
30.82	0.07	14.600	Cepheids	2013AJ....146...86T				74.4	
30.20	0.20	11.000	SNIa	2000A&A...361...63T	MLCS	SN 1999by		65	
30.39	0.12	12.000	SNIa	2002ApJ...568..791H		SN 1999by			
30.82	0.07	14.600	SNIa	2013AJ....146...86T		SN 1999by		74.4	
31.16	0.07	17.000	SNIa	2007ApJ...659..122J	MLCS2k2	Sn 1999by	0.002128	65	
31.16	0.15	17.100	SNIa	2001AJ....121.3127V	MLCS	SN 1999by		65	
30.38		11.900	Grav. Stability Gas. Disk	1996AstL...22...71Z					
29.54	0.04	8.070	HII region diameter	1982AN....303..329O	Photographic, 463 'dark cloud' diameters				
30.50	0.80	12.600	IRAS	1997ApJS..109..333W	A82, H			75	
30.25		11.200	Tully-Fisher	1985A&A...153..125G	B			90	
30.39	0.40	12.000	Tully-Fisher	1988NBGC.C....0000T	B			75	
30.54	0.17	12.800	Tully-Fisher	1984A&AS...56..381B	B			103	
30.54	0.17	12.800	Tully-Fisher	1986A&A...156..157B	B			103	
30.55	0.19	12.900	Tully-Fisher	1985A&AS...59...43B	B			103	
30.66	0.40	13.600	Tully-Fisher	1981ApJ...248..408D	B			100	
30.66	0.16	13.600	Tully-Fisher	1984A&AS...56..381B	Mean of B mag and Diameter TF relation			103	
30.66	0.17	13.600	Tully-Fisher	1985A&AS...59...43B	Mean of B mag and Diameter TF relation			103	
30.82	0.07	14.600	Tully-Fisher	2013AJ....146...86T				74.4	
30.89	0.40	15.100	Tully-Fisher	1985A&AS...59...43B	Diameter			103	
30.92	0.38	15.300	Tully-Fisher	1984A&AS...56..381B	Diameter			103	
30.94	0.20	15.400	Tully-Fisher	2002ApJ...565..681R	B				
30.96	0.30	15.600	Tully-Fisher	1983ApJ...265....1A				80	
31.30	0.30	18.200	Tully-Fisher	1994ApJ...430...53P	SN 1957A host				18.45
31.32	0.35	18.400	Tully-Fisher	2009AJ....138..323T					
31.48	0.36	19.700	Tully-Fisher	2009ApJS..182..474S	Malmquist cor.				
31.49	0.36	19.900	Tully-Fisher	2009ApJS..182..474S					
31.67	0.40	21.600	Tully-Fisher	2012ApJ...749..174C	HOSTS SN 1999by			75	
31.67	0.40	21.600	Tully-Fisher	2012ApJ...749..174C	HOSTS SN 1999by			75	

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RAR analysis

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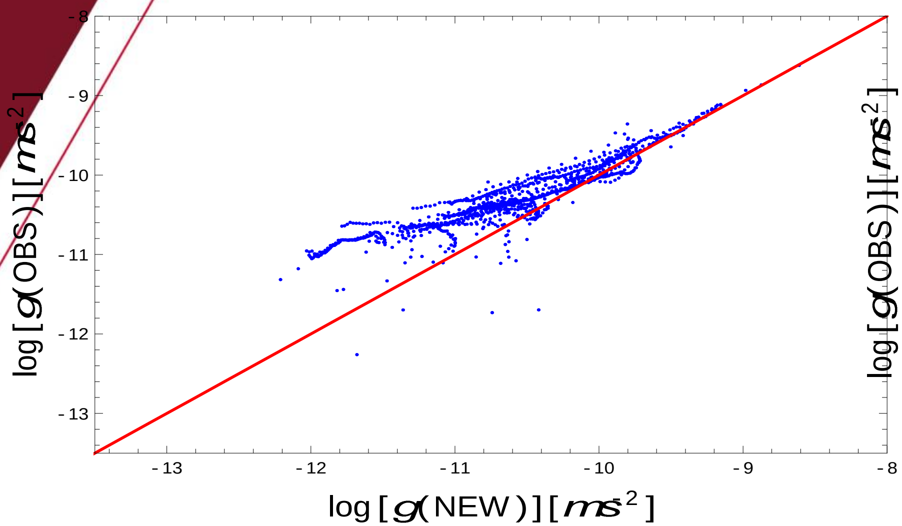
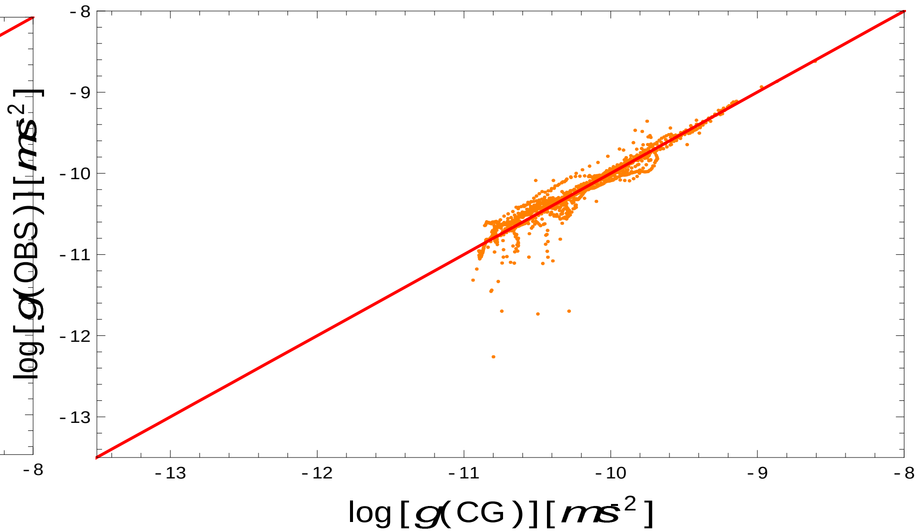
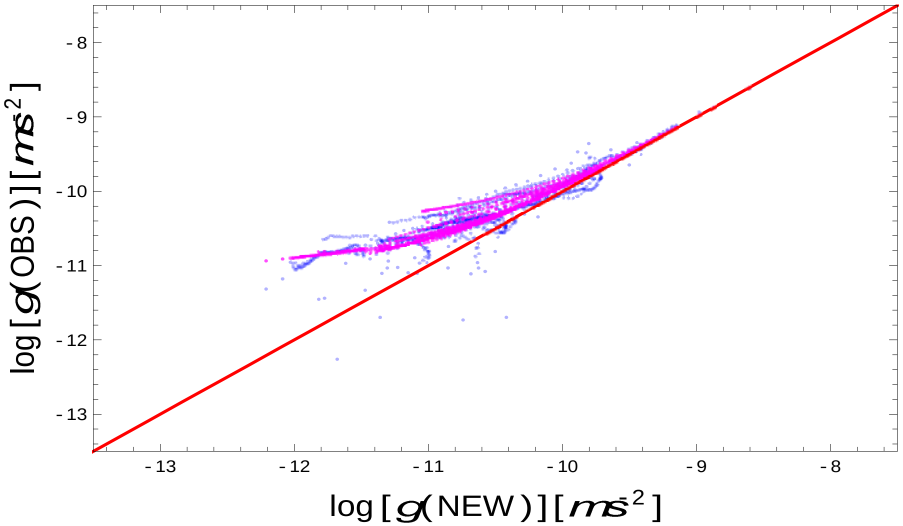
RAR using CG







Blue=All data points RAR with CG mass
Purple=CG prediction overlaid on the RAR
Orange=CG as the predictor for the baryons (no dark matter)

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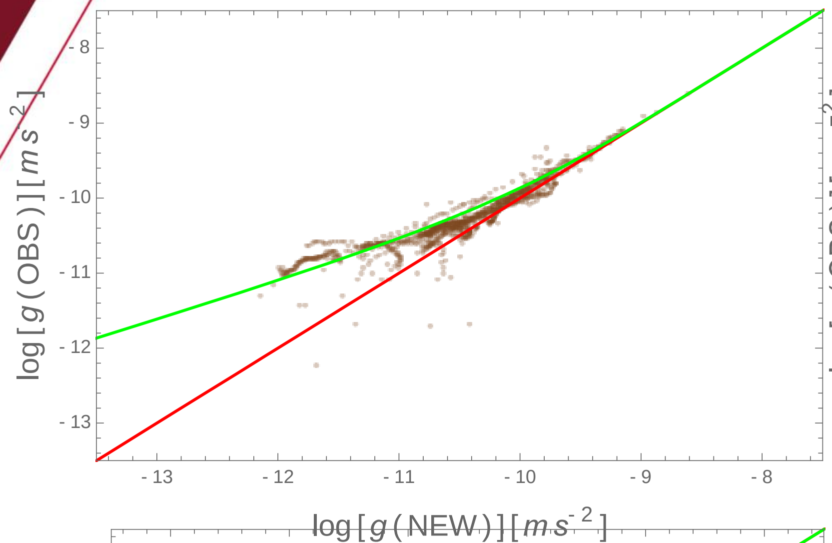
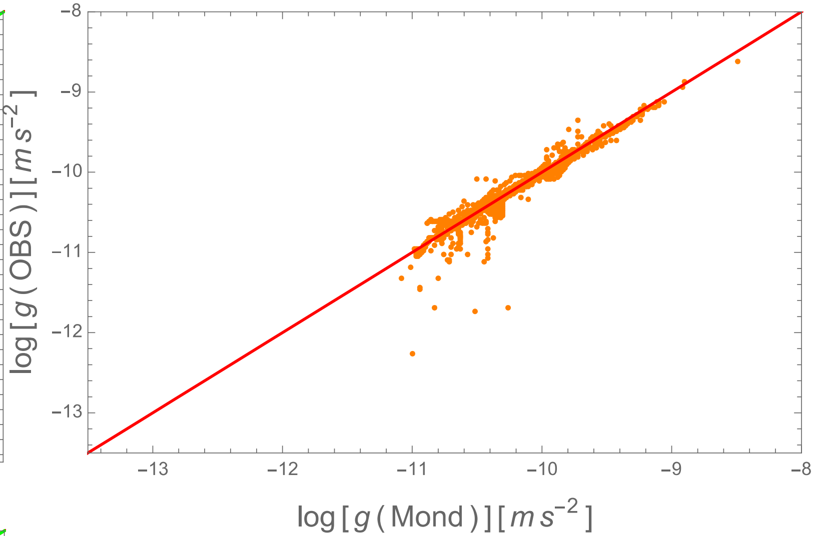
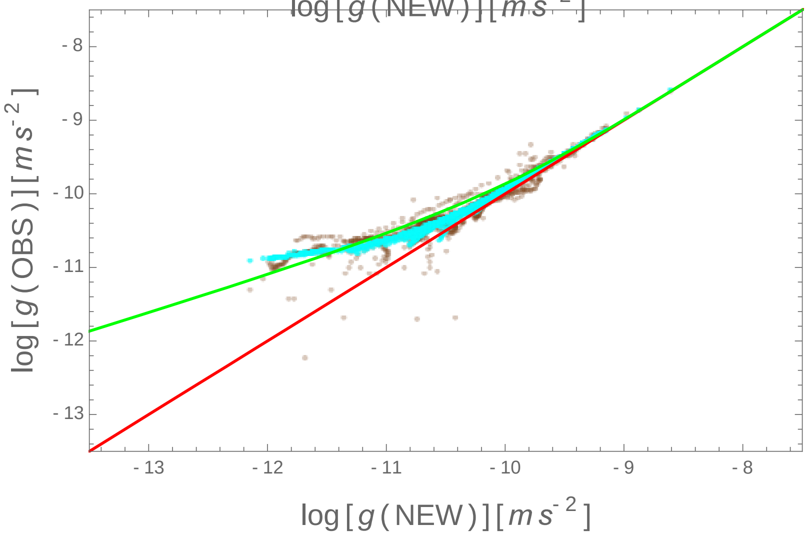
RAR in MOND







Brown=All data points
RAR for MOND mass
Purple=MOND prediction
overlaid on the RAR
Orange=MOND as the
predictor for the baryons
(no dark matter)
Green line=MLS

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Conclusions on RAR

- Both MOND and CG can capture the RAR without the need for dark matter
- Fitting function described by MLS has a free parameter, and even using MOND masses is not a correct line of best fit.
- When fit with NED distance, we have shown the MLS parameter can change by up to a factor of four in previous work using a larger data set.

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Overall Conclusions

- Using latest distance estimates, both MOND and CG can account for the two surveys quite well in Rotation curves, TF and RAR.
- TF should only be used as a test, not a given, and CG now allows for deviations to be shown in the data.
- The initial RAR formalism does not completely align with CG or MOND, but both can give a fundamental prediction for RAR without the need for dark matter.

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Future Work

- Comparisons of MLS, CG, MOND predictions on more individual samples: (see James G. O'Brien *et al* 2018 *ApJ* **852** 6)
- Posit CG as a test to remove circular argument of TF for distance estimates. Code writing for this is already underway

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Thank you

- Questions?

References:

[1] S. S. McGaugh, F. Lelli, and J. M. Schombert, Phys. Rev. Lett. **117**, 201101 (2016).

[2] M. Milgrom, Ap. J. **270**, 365, 371, 384 (1983).

[3] P. D. Mannheim and J. G. O'Brien, Phys. Rev. Lett. **106**, 121101 (2011); Phys. Rev. D **85**, 124020 (2012); Mon. Not. Roy. Astron. Soc. **421**, 1273 (2012); J. Phys. Conf. Ser. **437**, 012002 (2013).

[4] S.-H. Oh et. al., A. J. **149**, 180 (2015).

[5] P. D. Mannheim and D. Kazanas, Ap. J. **342**, 635 (1989).

[6] F. Lelli, S. S. McGaugh, J. M. Schombert, and M. S. Pawloski, Ap. J. **836**, 152 (2017).

[7] J. W. Moffat and S. Rahvar, Mon. Not. Roy. Astron. Soc. **436**, 1439 (2013).

[8] S. S. McGaugh, J. M. Schombert, G. D. Bothum, and W. J. G. de Blok, Ap. J. Lett. **533**, 99 (2000).

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