Particle Appearance and Disappearance

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May 25, 2020

Particle Stability in Relativistic Quantum Mechanics

Purpose of the paper:
Challenge a common misconception

- The common view today
  - Quantum mechanics cannot model particle disappearance
    - Neither non-relativistic nor relativistic formulations
  - QFT must be used
- Goal: show by example that the common view is incorrect
- Use parametrized Relativistic Quantum Mechanics (pRQM) as the example

Introduction:
How did the misconception arise?

Do particles exist forever?
Non-Relativistic View

Schroedinger Equation

$$i\hbar \frac{\partial \psi_S}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi_S + V \psi_S$$

Positive Definite Probability Density

$$\rho_S = \psi_S{}^* \psi_S \geq 0$$

Normalization over space

$$\int \rho_S d^3x = 1$$

⇒ A particle exists somewhere for ALL time

Do particles exist forever?
The Experimental View

- **Particles can appear and disappear**
- **Observations**
  - **Exponential decay law**
  - **Particle creation and annihilation**
  - **Mass state transitions**
  - **Mass-energy transformation**
  - **Mass-energy uncertainty principle**

$\Rightarrow$ A particle MAY DISAPPEAR at a given time

Do particles exist forever?
RQM* View

Single particle relativistic wave equations, e.g.
Klein-Gordon (below), Dirac, Proca, etc.

$$\left[\frac{\hbar}{i}\frac{\partial}{\partial x_\mu} - \frac{e}{c}A^\mu\right]\left[\frac{\hbar}{i}\frac{\partial}{\partial x^\mu} - \frac{e}{c}A_\mu\right]\Psi = m_0^2 c^2 \Psi$$

Assumes particles are fundamental

***RQM = Relativistic Quantum Mechanics**

Traditional Issues with the RQM View

- How do we interpret …
  - Negative energy solutions
  - Negative probability densities
- Dirac's hole theory, aka Dirac Sea
  - Works for fermions but not bosons
- Particle creation and destruction
  - Single particle formulation inadequate?

Wave-Particle Duality

- Consider photon-electron example
  - Treat photons and electrons on equal footing
- Suppose particles are fundamental
  - Electromagnetic field arises from collection of photons
- Suppose fields are fundamental
  - Photon arises from quantization of electromagnetic field
  - Electron arises from quantization of matter field

Particles or Fields?

- **Particles are fundamental in RQM**
  - **Eigenfunction Ψ is probability of seeing a particle at some location**
- **Fields are fundamental in QFT**
  - **Eigenfunction Ψ is a field**
  - **Ψ is probability of seeing a particular configuration of a field**
  - **Particles arise after quantization of fields**
- **$\Rightarrow$ Particles are excitations of fields**
  - **Excitations can appear and disappear**

Do particles exist forever?
QFT View – Peskin and Schroeder ca. 1995

"Why can't we just quantize relativistic particles the way we quantized nonrelativistic particles?" [Peskin and Schroeder, pg. 13, 1995]

"We have no right to assume that any relativistic process can be explained in terms of a single particle, since the Einstein relation $E = mc^2$ allows for the creation of particle-antiparticle pairs. Even when there is not enough energy for pair creation, multiparticle states appear…" [Peskin and Schroeder, pg. 13, 1995]

Peskin, M.E. and D.V. Schroeder, An Introduction to Quantum Field Theory, Taylor and Francis: Boca Raton, Florida

Do particles exist forever?
QFT View – Wilczek ca. 1998

Noninteracting QFT implies "the existence of indistinguishable particles, the phenomenon of quantum statistics, and the existence of antiparticles" [Wilczek, pg. 2, 1998]

Interacting QFT implies "the ubiquity of particle creation and destruction processes." [Wilczek, pg. 3, 1998]

Wilczek, F., http://arxiv.org/abs/hep-th/9803075v2, 19 May 1998; see also APS Centenary Issue of Reviews of Modern Physics, March 1999.

Do particles exist forever?
QFT View – D. Tong ca. 2006

"…the combination of quantum mechanics and special relativity implies that particle number is not conserved." [Tong, pg. 2, 2006]

"There is no mechanism in standard non-relativistic quantum mechanics to deal with changes in particle number." [Tong, pg. 3, 2006]

"…once we enter the relativistic regime we need a new formalism in order to treat states with an unspecified number of particles. This formalism is quantum field theory" [Tong, pg. 3, 2006]

Tong, D., 2006, Quantum Field Theory, Univ. of Cambridge Part III Lectures, http://www.damtp.cam.ac.uk/user/tong/qftvids.html accessed 1/23/20

How are QFT constructed?
[Wilczek, pg. 4, 1998]

- Specify a continuum field theory (CFT)
  - Including Poisson brackets
- Apply rules of quantization to CFT
  - Replace Poisson brackets
  - Use commutators for bosonic fields
  - Use anticommutators for fermionic fields
- Standard Model is a QFT
  - Associate a field with each fundamental particle
- QFT procedure can be used with parametrized theories [e.g. Pavsic, 2001]

Pavsic, M, 2001, The Landscape of Theoretical Physics, Kluwer Academic, Dordrecht

Wilczek, F., http://arxiv.org/abs/hep-th/9803075v2, 19 May 1998; see also APS Centenary Issue of Reviews of Modern Physics, March 1999.

Can issues with the RQM View be resolved?

- **Negative energy solutions**
  - **Stueckelberg-Feynman interpretation**
- **Negative probability densities**
  - **Positive probability densities**
  - **Extend inner product over space-time**
- **Dirac’s hole theory, aka Dirac Sea**
  - **Not needed in parametrized formulation**
- **Particle creation and destruction**
  - **Provide multi-particle parametrized formulation**

parametrized Relativistic Quantum Mechanics

Relativistic Dynamics

- **Relativistic Dynamics**
  - **Theories with invariant evolution parameter**
- **Many formulations**
  - **e.g. review in Fanchi, *Found. Phys. 23*, 487 (1993)**
- **Relativistic Dynamics model used here**
  - **Fanchi, *Parametrized Relativistic Quantum Theory* (Kluwer, Dordrecht, 1993) and references therein**
  - **Horwitz, *Relativistic Quantum Mechanics* (Springer, 2015)**

Conventional Postulates For Quantizing Classical Theory

P1. The states of a physical system are characterized by unit rays in a normed, linear vector space (Hilbert space)

P2. Superposition applies: if two states or system are represented by ψ_1 and ψ_2, then $\Psi = a_1 \psi_1 + a_2 \psi_2$ is also a state of the system.

P3. Associate a Lie algebra with a set of transformations.

- **Physical measurements are associated with generators of transformations**
- **Each physical observable is represented by a linear Hermitian operator**

New Postulate For Quantizing Classical Theory*

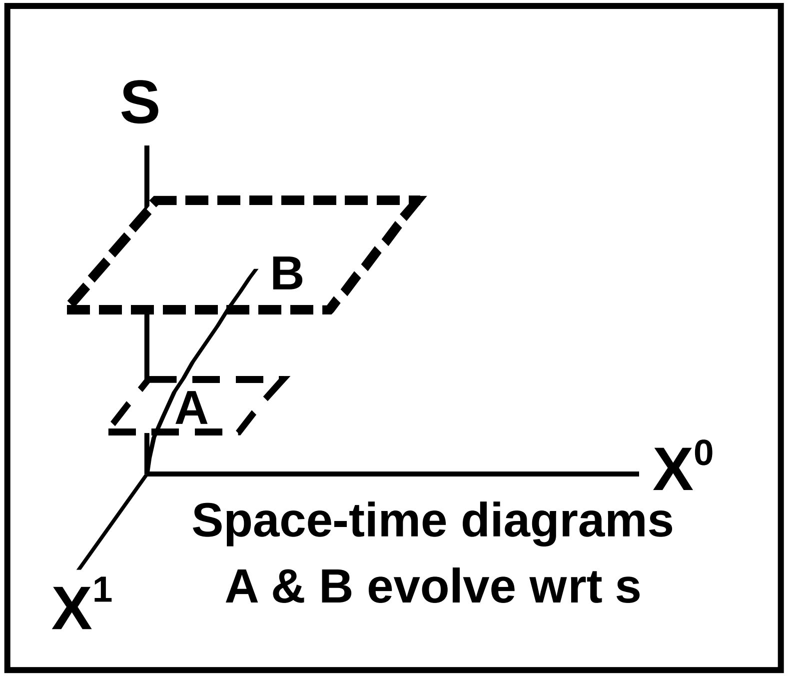
P4. The dynamical evolution of the system is given by a unitary operator that is a function of a scalar evolution parameter.

*** SHP (Stueckelberg-Horwitz-Piron) formulation of postulates, Stueckelberg (1941-42); Horwitz and Piron (1973)**

Resolve Temporal Enigma

Two "Times"

- **Minkowski time**
  - **temporal coordinate of a space-time four-vector**
  - **aka Einstein's Time**
- **Historical time**
  - **ordering parameter of PRQT**
  - **aka Newton's Time**



Path Integral Formulation

Sum the Probability Amplitudes for all possible paths

Calculate amplitude at space-time x_{i+1}, t_{i+1} and evolution parameter s+ε from amplitude at x, t and s

Define Probability Amplitude in terms of Action

$$\phi(x_{i+1}, t_{i+1}, s+\varepsilon) = \frac{1}{A}\int e^{iS(x_i,t_i)}\phi(x_i, t_i, s)\,dx_i\,dt_i$$

Define Action in terms of Lagrangian

$$S(x_i, t_i) = \int L(\dot{x}_i, \dot{t}_i)\,ds$$

Path Integral – Free Particle

Lagrangian for free particle

$$L(\dot{x}, \dot{t}) = \frac{m}{2}\left[\left(\frac{dt}{ds}\right)^2 - \left(\frac{dx}{ds}\right)^2\right]$$

Action for free particle

$$S(x_i, t_i) = \varepsilon \frac{m}{2}\left[\left(\frac{t_{i+1} - t_i}{\varepsilon}\right)^2 - \left(\frac{x_{i+1} - x_i}{\varepsilon}\right)^2\right]$$

Construct Stueckelberg Equation for free particle

$$\frac{\partial \phi}{\partial s} = \frac{i\hbar}{2m}\left[\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2}\right]$$

Klein-Gordon Equation for Free Particle

Stueckelberg Equation for free particle

$$i\hbar\ \frac{\partial}{\partial s}\ \psi(x,s) = -\ \frac{\hbar^2}{2m}\ \partial_\mu \partial^\mu \psi(x,s)$$

Stationary state solution

$$\psi(x,s) = \psi_{KG}(x) exp\left[-i\,\frac{M^2 s}{2m\hbar}\right]$$

Klein-Gordon equation

$$M^2 \psi_{KG} = -\ \hbar^2\ \partial_\mu \partial^\mu \psi_{KG}$$

Probabilistic Basis of parametrized Relativistic Quantum Mechanics

Probability Concepts in Space-Time

Positive Definite Probability Density

$$\rho(x|s) = \Psi * (x,s)\Psi(x,s) \geq 0$$

Normalization over space-time

$$\int_D \rho(x|\mathrm{s}) d^4x = 1$$

Continuity Equation

$$\frac{\partial \rho}{\partial s} + \partial_\mu(\rho V^\mu) = 0$$

Single Particle Quantum Formalism

Assume Decomposition

$$\Psi(x,s) = \sqrt{\rho(x|s)}\, e^{i\xi(x,s)}$$

Velocity Four-Vector

$$V^{\mu}(x,s) = \frac{\hbar}{m}\frac{\partial\xi(x,s)}{\partial x_{\mu}} - \frac{e}{mc}A^{\mu}(x,s)$$

Probability Flux

$$\rho V^{\mu} = -\frac{i\hbar}{2m}\left[\Psi * \frac{\partial\Psi}{\partial x_{\mu}} - \Psi\frac{\partial\Psi *}{\partial x_{\mu}}\right] - \frac{eA^{\mu}}{mc}\Psi * \Psi$$

Expectation Value

Expectation value of observable Ω

$$\langle \Omega \rangle = \int \Psi * \Omega \Psi dx$$

where Ψ(x,s) $\in$ L^2(x)

Uncertainty Principle in Space-Time

$$|\Delta x_\mu| \, |\Delta p_\mu| \geq \frac{\hbar}{2}$$

(No summation here)

Single Particle Stueckelburg Equation

$$i\hbar \frac{\partial \Psi}{\partial s} = \frac{\pi^{\mu}\pi_{\mu}}{2m}\Psi + V\Psi$$

where

$$\pi^{\mu} = \frac{\hbar}{i}\frac{\partial}{\partial x_{\mu}} - \frac{e}{c}A^{\mu}$$

Free Particle Stueckelburg Equation

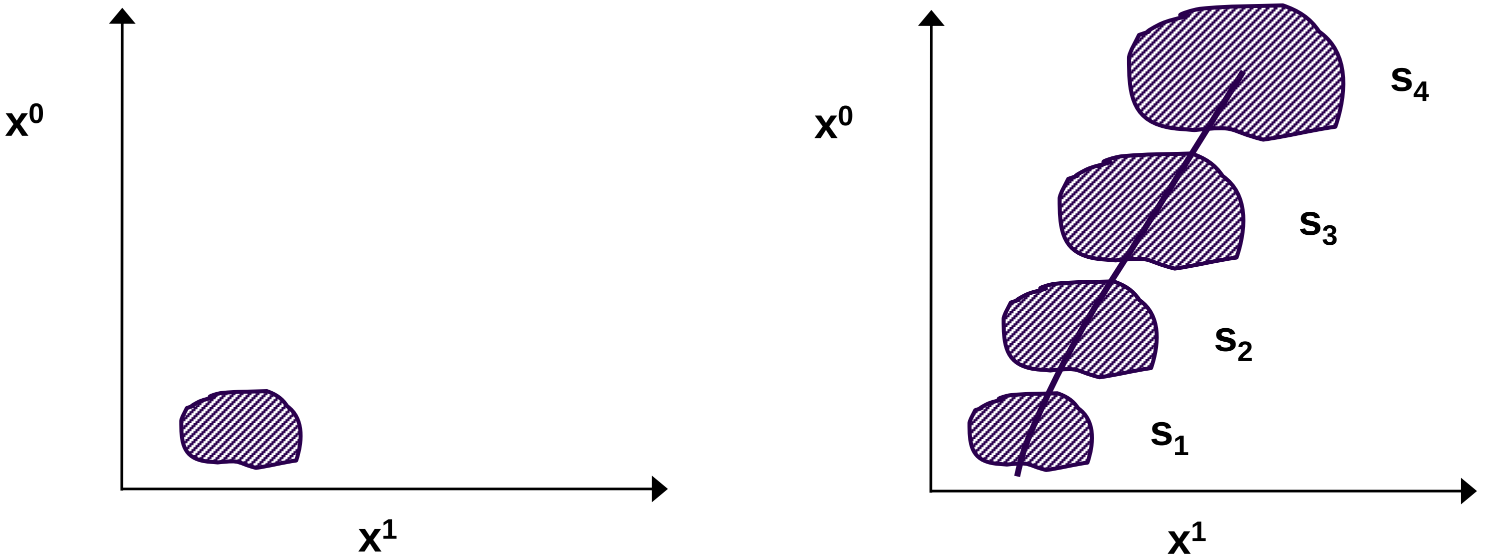
$$i\hbar \frac{\partial}{\partial s} \psi(x,s) = -\frac{\hbar^2}{2m} \partial_\mu \partial^\mu \psi(x,s)$$

On-Shell Mass

$$m^2c^2 = \frac{\left\langle p_f^\mu p_{f\mu} \right\rangle}{sgn\left\{\delta \left\langle x_f^\mu \right\rangle \delta \langle x_{f\mu} \rangle \right\}} = \frac{\hbar^2 \left\langle k_f^\mu k_{f\mu} \right\rangle}{sgn\left\{\delta \left\langle x_f^\mu \right\rangle \delta \langle x_{f\mu} \rangle \right\}} \geq 0$$

Applicable to both bradyons and tachyons

Wave Packet Evolving in Space-time



a. Localized Wave Packet

b. World line

N-Body Stueckelburg Equation

$$i\hbar\frac{\partial\underline{\Psi}}{\partial s}=\underline{K_N}\underline{\Psi}=\left\{\sum_{a=1}^{N}\frac{\pi_a^{\mu}\pi_{a\mu}}{2m_a}\underline{I}+\underline{V}\right\}\underline{\Psi}$$

With N-body mass operator K_N and operators

$$\pi_a^{\mu}=p_a^{\mu}-\frac{e}{c}A_a^{\mu} \qquad p_a^{\mu}=\frac{\hbar}{i}\frac{\partial}{\partial x_{a\mu}}$$

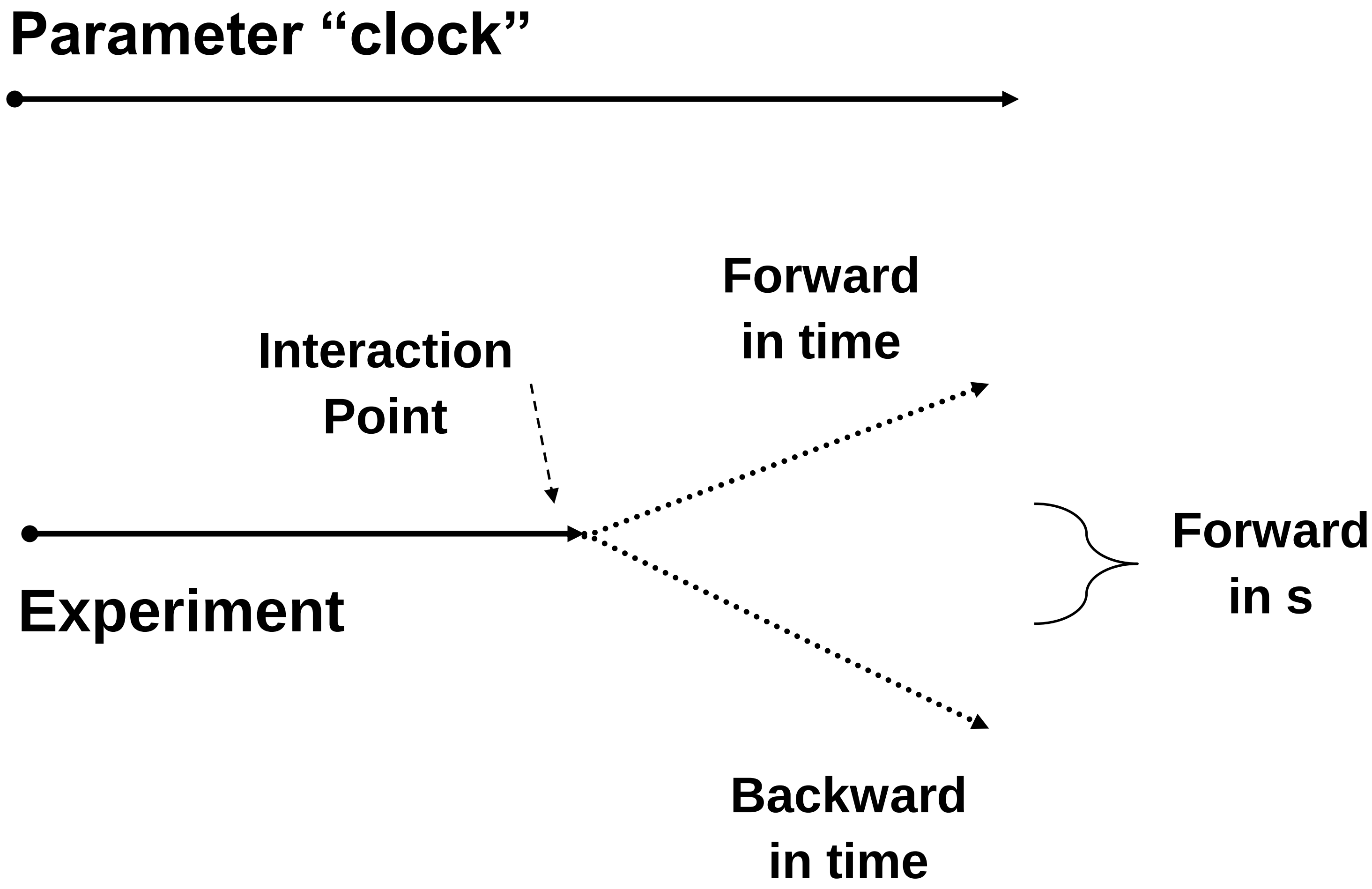
Expectation Value of Observable

$$\langle\Omega\rangle=\int\underline{\Psi}^{+}\Omega\underline{\Psi}dx$$

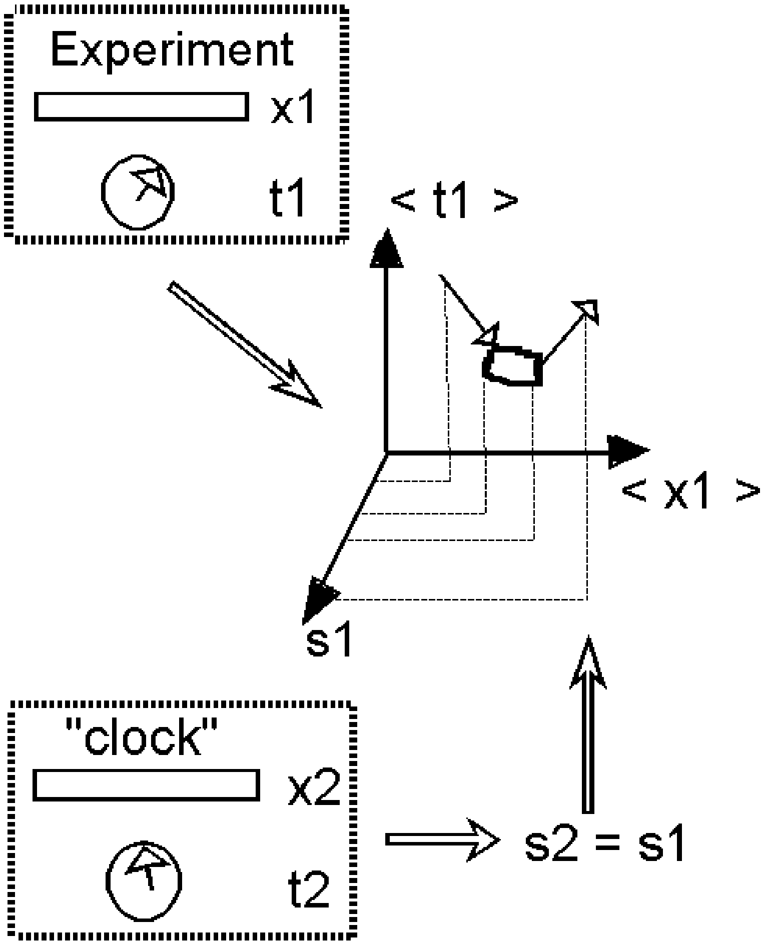
On-Shell Mass of Free Particle a

$$m_a^{\mu}c^2=\left\langle p_{fa}^{\mu}p_{fa\mu}\right\rangle=\hbar^2\left\langle k_{fa}^{\mu}k_{fa\mu}\right\rangle$$

Measuring the Evolution Parameter



Evolution Parameter Clock



Illustrative Applications of pRQM to Particle Appearance and Disappearance

pRQM applications imply experimentally testable predictions

Exponential Decay

Empirical particle decay

$$N(t) = N(0)e^{-\lambda t}$$

where $N(t')$ = number of particles observed at time t' and
λ = constant probability of particle decay per unit time

The probability of observing a particle at time *t* is

$$P_{obs} = \frac{N(t)}{N(0)} = e^{-\lambda t}$$

***Fanchi, Parametrized Relativistic Quantum Theory [1993]**

pRQM Model of an Unstable Particle* – 1

pRQM model of a single unstable particle with mass m

Stückelberg equation with $\hbar = c = 1$

$$i \frac{\partial \Psi(x,s)}{\partial s} = \left[\frac{\pi^{\mu}\pi_{\mu}}{2m} + V_I\right]\Psi(x,s)$$

Normalization Condition

$$\int \rho(x|s)d^4x = 1$$

***Fanchi, Parametrized Relativistic Quantum Theory [1993]**

pRQM Model of an Unstable Particle – 2

Assume mass state is stationary so that $\rho(x|s) \rightarrow \rho(x)$, thus

$$\Psi(x,s) = \psi(x)e^{-iM^2 s/2m}$$

Resulting Stückelberg equation for stationary mass state is

$$M^2\psi\ (x,s) = [p^\mu p_\mu + U_I]\ \psi\ (x,s)$$

with interaction operator U_I

$$U_I = -\ eA^\mu p_\mu -\ ep^\mu A_\mu + e^2 A^\mu A_\mu + 2mV_I$$

***Fanchi, Parametrized Relativistic Quantum Theory [1993]**

pRQM Model of an Unstable Particle – 3

Assume U_I satisfies adiabatic condition

$$U_I \psi(x) = u_I \psi(x)$$

where u_I is approximately constant in region of interest

Solution

$$\Psi(x,s) \sim exp\left[-i\ \frac{M^2 s}{2m} - \frac{\lambda}{2}\ t + i\vec{k} \cdot \vec{x} \right]$$

The state $\Psi(x,s)$ decays in time but does not propagate in time

∴ Marginal probability density in time has exponential decay form

***Fanchi, Parametrized Relativistic Quantum Theory [1993]**

Interference in Time

- **Predicted: Horwitz and Rabin, Lettere Nuo. Cim 17, 501 (1976)**
- **Possible experimental confirmation: Lindner, et al. experiment [PRL 95 (2005) 040401] discussed by Horwitz, Phys. Lett. A355, 1 (2006)**

K-mesons and Tachyons

- **K-meson model**
  - **Fanchi, Found. Phys. 33, 1189 (2003)**
- **Tachyon kinematics**
  - **Tachyons created and destroyed in relativistic quantum tunneling**
  - **Fanchi, Found. Phys. 20, 189 (1990)**

Neutrino oscillation models

- **Transition between mass states**
  - **Fanchi, Found. Phys. 28, 1521 (1998)**
  - **Fanchi, Horizons in World Physics, Vol. 240, 117 (2003)**
  - **Fanchi, J. Phys.: Conf. Ser. 845, 012027 (2017)**
  - **Fanchi, Horizons in World Physics, Vol. 300, 205 (2019)**
- **Rusov and Valenko analysis**
  - **J. Phys.: Conf. Ser. 361, 012033 (2012)**
  - **Estimated neutrino mass from Fanchi model < 0.2 eV**
    - **Mass just below minimum direct mass (0.2 eV) observable at KATRIN**
  - **Shows difference between parametrized and conventional models**

Conclusions

- **Common misconception**
  - **Claim that quantum mechanical theories cannot model particle disappearance**
- **Counterexample**
  - **pRQM is applicable to particle appearance and disappearance**
- **pRQM is**
  - **a manifestly covariant quantum theory**
  - **an extension of the standard paradigm**
    - **adds invariant evolution parameter**
    - **defines expectation value over all space-time**

Thank you!

Send questions or comments to

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